Pricing electricity derivatives: A mean-reverting and seasonal approach

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Abstract

This paper evaluates the (in- and out-of-sample) empirical performance of some models that incorporate mean-reversion and seasonality in relation to electricity. In more detail, we compare different specifications of the Moreno, Novales, and Platania (2019) model and those proposed in Schwartz (1997) and Lucia and Schwartz (2002). We use electricity daily futures prices from 01/07/2002 to 01/06/2023. Our main finding from the in-sample estimation is that models that include Fourier terms to account for different types of seasonality are superior to our chosen benchmark models, however these results do not extend to the out-of-sample analysis.

Keywords: Electricity futures; mean-reversion; Fourier series; long-term swings, seasonality.

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1 Introduction

For a large portion of history renewable energy was the only energy source people had. That was until the mid 1800s ushered in the Industrial Revolution, when coal and other fossil fuels gained popularity and became the most common source of energy (Karekezi and Kithyoma, 2003). However, in the past two decades renewable energy regained its importance. This is mainly due to the wider population becoming increasingly aware of the dangers posed by climate change and aiming for net-zero emissions, but also due to cost savings. The more renewable energy technologies are deployed, the cheaper they become to implement and maintain. Power supplies such as wind, solar and hydro are crucial in the effort to reach the carbon-neutrality (De La Peña *et al.*, 2022). This shift towards a cleaner and more sustainable energy future has introduced new dynamics and complexities to the electricity sector (Zou *et al.*, 2021). As these natural fuels heavily depend on weather conditions and cannot be stored (Letcher *et al.*, 2016), their management and pricing is posing a unique risk towards ensuring the stability of electricity markets.

To avoid a network collapse or an energy shortage, there is a need for an equilibrium between electricity production and consumption (Casula and Masala, 2021). Electricity futures and forwards are an essential tool in risk management and hedging strategies for all agents operating in electricity markets and therefore it is crucial that that their pricing accurately reflects the underlying conditions of the electricity market. (Boroumand *et al.*, 2015). By ensuring reliable and accurate valuation of electricity futures contracts, market participants can effectively hedge against price fluctuations and manage their exposure to volatile renewable energy generation.

Electricity is not a traditional commodity. It is non-storable and its transportation can be difficult, implying that it cannot be exchanged in different times because of the lack of inventories to respond to demand shocks. This feature also implies that the equilibrium of the system must be maintained continuously, and the production needs to reflect the demand (Ciarreta, Lagullón, and Zárraga, 2011). Regarding both issues, non-storability plays a significant role in pricing dynamics of these derivatives as it makes the price highly sensitive to real-time market demand, which can be influenced by multiple factors such as environmental conditions (time of the day, day in a week, season) and the business cycle (high production, low production) (Zhou *et al.*, 2016). Additionally, the limited capacity of building transmission lines and the possible electricity losses in these lines imply that the transport to certain regions can be very expensive or even impossible (Lucia and Schwartz, 2002).

We will now outline the key characteristics that define the behavior of the electricity sector.

- **Inelastic demand:** The intrinsic nature of electricity for our society implies that consumers generate an inelastic demand. When the electricity price increases, consumers could wait for a reasonable (lower) price and then consume it. Since electricity has become a first need asset for our economy (for both industries and families), consumers accept the offered price, despite the existing and potential increases. This characteristic is one of the reasons for the jumps in electricity prices that will be discussed now.

- **Jumps:** One of the main differences between prices of electricity and the rest of commodities is the presence of jumps in the electricity market. A jump appears if, in a short time period, the price increases rapidly to revert back later to another level (Volpe, 2009). These jumps have been analyzed by different models that, in general, include a Poisson process. Such high volatility and frequent jumps make it necessary to implement risk management and increase use of derivatives. See, for instance, Johnson and Barz (1999), Knittel and Roberts (2005), Mount, Ning, and Cai (2006), and Escribano, Peña, and Villaplana (2011).

- **Possibility of negative prices:** This is strongly related to the type of electricity generator. Some generators have high startup and shutdown costs as, for instance, the natural gas steam turbine powered by coal or nuclear power plants. As generators want to profit from high electricity prices, they can produce electricity even when the spot price is not high. As the competition among energy producers can be very high, we can find offers with negative prices .

- **Mean-reversion:** As the remaining commodities, electricity is assumed to converge to a certain long-term value, a consequence of a non-linear long-term trend pattern that can be independent of short-term seasonal cycles, see Bhanot (2000), Karesen and Husby (2000), Lucia and Schwartz (2002), and Knittel and Roberts (2005), among others. This reversion is also a consequence of changes in the demand that can increase prices, implying economic incentives for the entry of expensive generators in the electricity supply (Escribano, Peña, and Villaplana, 2011) and because the climate time evolution follows a mean-reverting process, implying that both the demand and equilibrium prices are affected (Knittel and Roberts, 2005).

- **Seasonality:** This is one of the main features in electricity market prices and it can be intraday, weekly, monthly, quarterly, or yearly. It seems reasonable to justify this seasonality in the dependence of the electricity consumption to industrial hours (electricity consumption is higher when the production process starts, daily or weekly, and it decreases along the week- ends) and to the yearly seasons. This seasonality has been shown in Bhanot (2000), Lucía and Schwartz (2002), León and Rubia (2004), and Arango and Larsen (2011), among others. Escribano, Peña, and Villaplana (2011) also reflect weekly and monthly seasonalities by using dummy variables and / or sinusoidal functions.

Considering unique electricity features mentioned above, our study is based on the model proposed in Moreno, Novales, and Platania (2019) (Moreno *et al.* (2019), from now on). This model assumes that commodity prices show mean-reversion and seasonality. Both features are characterized by Fourier series, allowing these authors to distinguish between long-term mean-reversion, as well as short- and medium-term seasonality. Specifically, we will extend this pricing model by adding terms to the Fourier series with the aim to enhance the model's ability to accurately capture the dynamics of electricity. We will empirically compare the performance of several particular cases of the Moreno *et al.* (2019) model and the specifications proposed in Schwartz (1997) and Lucia and Schwartz (2002). We will do so by analyzing both their in- and out-of-sample performance by applying non-linear least-squares and the Kalman filter, respectively. We will use futures contracts, the most exchanged-traded derivatives for electricity.

This paper is organized as follows. Section 2 provides a literature review to justify the adequacy of the models that include mean-reversion and seasonality for electricity. Section 3 briefly describes the models under analysis and Section 4 describes the statistical methodology that will be used to estimate these models and the futures prices that we have used. Section 5 includes the (in- and out-of-sample) empirical analysis of all the models and discusses the results. Finally, Section 6 summarizes the main conclusions and suggests some possible lines of future research.

2 Literature review

Fama and French (1987) proposed the expectation theory for pricing commodity futures contracts. According to this theory, futures price for a commodity depends on two variables; the expected spot price and the risk premium. Expected spot price is what market participants believe the price of the commodity will be at the delivery, and risk premium is what producers require for bearing the uncertainty of the delivery price against the market price. This is the main starting point for a majority of futures contracts pricing models, including the three models we will be introducing later.

This Section provides a (non-exhaustive) review of the papers that have proposed and analyzed mean-reversion and seasonality in modeling commodity prices with special emphasis on electricity.

Mean-reversion indicates a trend in asset prices to converge to a certain long-term level. The intuition is that, when an asset price is higher than the convergence level, the supply of the asset will increase, as the players with higher production cost enter the market. This will ultimately lead to a fall in the commodity price. Similarly, low spot price will follow decrease is supply, which will put an upward lift to the price. Lutz (2010) discussed mean-reversion in asset prices highlighting the correlation between spot prices and the convenience yield. Convenience yield represents the additional and intangible benefit that the physical holder of commodity has, such as guaranteed supply, operational flexibility and quality assurance. It reflects the markets' expectations on the future availability of the commodity, see Hull (2021). This is in line with the Kaldor and Working hypothesis, which indicates that the convenience yield depends inversely on the inventory level. Since mean-reversion seems to be empirically given for many commodities, a lot of them start with this assumption.

This property in energy assets has been analyzed for other commodities, see Gibson and Schwartz (1990), Schwartz (1997), and Schwartz and Smith (2000). Gibson and Schwartz (1990) analyzed crude oil forward contracts and considered a mean-reverting behavior in commodity prices, modeling the convenience yield as a second stochastic factor. Schwartz (1997) compared three stochastic mean-reverting models for commodity prices and provided empirical evidence of strong mean-reversion in commodity prices. Schwartz and Smith (2000) reformulated the Gibson and Schwartz (1990) model considering a latent convenience yield and proposed a two-factor model that allows mean-reversion in short-term prices and uncertainty in the long-term equilibrium price.

More examples of mean-reverting models can be found in Geman and Nguyen (2005) or Paschke and Prokopczuk (2010), among others. Besides, some models combine mean-reversion with other empirically observed characteristics, for example including jumps and seasonality as Cartea and Figueroa (2005) or stochastic volatility as Geman (2007).

Hylleberg (1992) defined seasonality as “the systematic, although not necessarily regular, intra-year movement caused by the changes of the weather, the calendar, and timing of decisions, directly or indirectly through the production and consumption decisions made by the agents of the economy”. This author indicates that these decisions are influenced by the expectations and preferences of the agents and the technology available in the economy. Thus, many commodities present seasonal fluctuations around their equilibrium price level. For example, agricultural assets can present seasonal variations ought to the seasonal nature of their supply and demand.

Seasonality is a common aspect taken into account by researchers when modeling the pricing of commodity derivatives. Many authors incorporate seasonality by considering its influence on returns and/or volatilities over time. Lucia and Schwartz (2002) captured the seasonality of electricity prices using a trigonometric deterministic function with a yearly period. Cartea and González-Pedraz (2012) introduced a deterministic long-term trend in the spot price process and Moreno and Platania (2015) and Moreno, Novales, and Platania (2018) used harmonic oscillators to capture time-varying fluctuations in interest rates. Moreno *et al.* (2019) proposed a two-factor model in which prices revert to a time-varying mean level, represented by a Fourier series, and included a second Fourier series in the price level.

Other authors have proposed time-varying conditional volatility models for electricity prices. For instance, León and Rubia (2004) proposed a ARIMA model for the spot electricity price with a GARCH specification for its volatility. Weron, Bierbrauer, and Trück (2004) used regime-switching models, Ciarreta, Lagullón and Zárraga (2011) incorporated time-varying volatility, and Escribano, Peña, and Villaplana (2011) suggested six models for different markets. Other relevant contributions are Arismendi *et al.* (2016), De Jong (2006), Geman and Roncoroni (2006), or Ewald and Zou (2021).

3 The Models

This section introduces all the one-factor models that will be used in our empirical analysis. We give a brief overview of the models found in the literature that study commodity prices, taking into account mean-reversion, seasonality, or a combination of both. These models will be analyzed empirically later.

3.1 Schwartz [1997]

This model incorporates mean-reversion into the calculation of commodity spot prices. It assumes that the commodity spot price St, at time t, is given by the following differential equation:

$$dS_t = \kappa \left(\mu - \ln\left(S_t\right)\right) S_t dt + \sigma S_t dW_t \tag{1}$$

where κ denotes the speed of mean-reversion, μ is the (long-term) mean-reversion value, σ is the diffusion coefficient, and W_t is a standard Wiener process. Note that the last term is stochastic while the previous ones are deterministic.

Applying the Itô's lemma to the logarithm of the spot price, $X_t = \ln\left(S_t\right)$, we get

$$d\ln\left(S_t\right) = \frac{1}{S_t} dS_t + 0 \cdot dt + \frac{1}{2}\left(-\frac{1}{S_t^2}\right)\left(dS_t\right)^2$$

Using equation (1), we have

$$\begin{aligned} d\ln\left(S_t\right) &= \kappa\left(\mu - \ln\left(S_t\right)\right) dt + \sigma dW_t - \frac{1}{2}\frac{1}{S_t^2}\sigma^2 S_t^2 dt \\ &= \kappa\left(\mu - \frac{\sigma^2}{2\kappa} - \ln\left(S_t\right)\right) dt + \sigma dW_t \end{aligned} \tag{2}$$

This author assumes that the risk market price λ is constant. Let $\widetilde{W}_t = W_t + \lambda t$ be a standard Wiener process under the risk-neutral measure $\widetilde{P}$. Then, equation (2) becomes

$$\begin{aligned} d\ln(S_t) &= \kappa\left(\mu - \frac{\sigma^2}{2\kappa} - \ln(S_t)\right)dt + \sigma d(\widetilde{W}_t - \lambda t) \\ &= \kappa\left(\mu - \frac{\sigma^2}{2\kappa} - \frac{\sigma}{\kappa}\lambda - \ln(S_t)\right)dt + \sigma d\widetilde{W}_t \end{aligned}$$

Then, under the measure $\widetilde{P}$, the log-spot price follows the (stochastic) differential equation

$$d\ln(S_t) = \kappa(\widetilde{\alpha} - \ln(S_t))\,dt + \sigma\widetilde{W}_t \tag{3}$$

where $\widetilde{\alpha} = \mu - \frac{\sigma^2}{2\kappa} - \frac{\sigma}{\kappa}\lambda$ is the (risk-neutral) long-term value of the log-spot price and $\widetilde{W}_t = W_t + \lambda t$ is a standard Wiener process under the measure $\widetilde{P}$. The solution of equation (3) is given by

$$\ln(S_u) = e^{-\kappa(u-t)}\ln(S_t) + \left(1 - e^{-\kappa(u-t)}\right)\widetilde{\alpha} + \sigma int_t^u e^{-\kappa(u-s)} d\widetilde{W}_s$$

Proof: Let $A(S_t, t) = e^{\kappa t}(\widetilde{\alpha} - \ln(S_t))$. Itô's lemma implies

$$\begin{aligned} dA(S_t,t) &= A_{S_t}dS_t + A_t dt + \frac{1}{2}A_{S_tS_t}(dS_t)^2 \\ &= -e^{\kappa t}\left(\kappa(\widetilde{\alpha} - \ln(S_t))\,dt + \sigma\widetilde{W}_t\right) + \kappa e^{\kappa t}(\widetilde{\alpha} - \ln(S_t))dt \\ &= -e^{\kappa t}\sigma d\widetilde{W}_t \end{aligned}$$

Integrating in $[t, u]$, we get

$$A(S_u, u) - A(S_t, t) = \sigma\int_t^u e^{\kappa s}\, d\widetilde{W}(s)$$

Then,

$$e^{\kappa u}(\widetilde{\alpha} - \ln(S_u)) = e^{\kappa t}(\widetilde{\alpha} - \ln(S_t)) + \sigma\int_t^u e^{ks}\, d\widetilde{W}(s)$$

Multiplying by $e^{-\kappa u}$ and rearranging terms concludes the proof.

This Proposition shows that the log-spot price follows a Gaussian distribution.

Given the filtration $\mathscr{F}_t$, the first two conditional statistical moments of the log-spot price are given as

$$\widetilde{\mathbb{E}}_t\left[\ln(S_T)\right] = \widetilde{\mathbb{E}}\left[\ln(S_T) \mid \mathscr{F}_t\right] = e^{-\kappa(T-t)}\ln\left(S_t\right) + \left(1 - e^{-\kappa(T-t)}\right)\widetilde{\alpha}$$
$$\widetilde{\mathbb{V}}_t\left[\ln(S_T)\right] = \widetilde{\mathbb{V}}\left[\ln(S_T) \mid \mathscr{F}_t\right] = \frac{\sigma^2}{2\kappa}\left(1 - e^{-2\kappa(T-t)}\right)$$

where the variance is computed by applying the isometry property for stochastic integrals. We can indicate two important and intuitive comments on both statistical moments:

- The expected value of the log-spot price is a weighted average of the current price $(\ln(S_t))$ and the long-term value $\widetilde{\alpha}$ at which this price converges. The weights are given by $e^{-\kappa(T-t)}$ and $1 - e^{-\kappa(T-t)}$. Then, when time goes by, the expected log-spot price moves from the current value and converges monotonically to the value $\widetilde{\alpha}$.

- The initial variance of the log-spot price is zero and it increases with the maturity of the futures. For long-term futures, this variance converges to $\frac{\sigma^2}{2\kappa}$, a (bounded) value that increases with the diffusion coefficient σ and decreases with the speed of mean-reversion κ.

Then, the spot price follows a lognormal distribution. The delivery price of a futures contract is the expected value of its underlying asset at a certain time later. Then, applying the properties of the lognormal distribution, the price of the commodity futures is given by

$$\begin{aligned} F\left(S_t, t, T\right) &= \widetilde{\mathbb{E}}_t\left[S_T\right] = \exp\left\{\widetilde{\mathbb{E}}_t\left[\ln\left(S_T\right)\right] + \frac{1}{2}\widetilde{\mathbb{V}}_t\left[\ln\left(S_T\right)\right]\right\} \\ &= \exp\left\{e^{-\kappa(T-t)}\ln\left(S_t\right) + \left(1 - e^{-\kappa(T-t)}\right)\widetilde{\alpha} + \frac{\sigma^2}{4k}\left(1 - e^{-2\kappa(T-t)}\right)\right\} \end{aligned}$$

Alternatively, the log-futures price is given by:

$$\ln\left(F\left(S_t, t, T\right)\right) = e^{-\kappa(T-t)}\ln\left(S_t\right) + \left(1 - e^{-\kappa(T-t)}\right)\widetilde{\alpha} + \frac{\sigma^2}{4k}\left(1 - e^{-2\kappa(T-t)}\right)$$

3.2 Lucia and Schwartz [2002]

These authors proposed a model that reflects a seasonal pattern in electricity market prices by means of dummy variables and trigonometric functions. In more detail, they suggested a simple sinusoidal function to capture the seasonal behavior of spot and forward electricity prices in the Nord Pool.

In short, they assume that the log-spot price can be split into two parts:

$$\ln(S_t) = f_t + Y_t$$

where f_t and Y_t are, respectively, the determninistic and stochastic parts.

In more detail, we have the following:

- The first component, f_t, is a deterministic function that models regularities in the evolution of prices, such as a deterministic trend and any periodic behaviour. This function is given by

$$f_t = \alpha + \beta D_t + \gamma \cos\left((t+\varphi)\frac{2\pi}{365}\right)$$

  where α, β, γ, and φ are constants, and the dummy variable D_t takes value one if date t is week-end or holiday, or zero otherwise

- The second component, Y_t, is stochastic and it is assumed to follow a particular continuous-time diffusion process, given by

$$dY_t = -\kappa Y_t dt + \sigma dW$$

  where κ denotes the speed of mean-reversion, σ is the diffusion coefficient, and W is a standard Wiener process.

Thus, under the risk-neutral measure $\widetilde{P}$, considering the risk market price λ as constant, Y_t is given by the following stochastic process:

$$dY_t = -\kappa(\widetilde{\alpha} - Y_t)dt + \sigma d\widetilde{W}$$

where $\widetilde{\alpha} = -\frac{\sigma}{\kappa}\lambda$ and $\widetilde{W}_t$ is a (risk-neutral) Wiener process.

Under the measure $\widetilde{P}$, in a similar way to Schwartz (1997), it is obtained that

$$\ln(S_T) = f_T + Y_t e^{-\kappa(T-t)} + \tilde{\alpha}\left(1 - e^{-\kappa(T-t)}\right) + \sigma \int_t^T e^{\kappa(s-t)} d\tilde{W}(s)$$

Computing the conditional statistical moments of this log-spot price, using $F(S_t, t, T) = \widetilde{\mathbb{E}}_t[S_T]$, and applying properties of the lognormal distribution, the expression for the log-futures price is:

$$\ln(F(S_t, t, T)) = f_T + e^{-\kappa(T-t)}(\ln(S_t) - f_t) + \left(1 - e^{-\kappa(T-t)}\right)\widetilde{\alpha} + \frac{\sigma^2}{4k}\left(1 - e^{-2\kappa(T-t)}\right)$$

3.3 Moreno, Novales, and Platania [2019]

These authors proposed a pricing model which captures not only a short-term seasonal component but also what they name long-term swings. These long-term swings represent long-term fluctuations around a mean-reversion level, a feature that is empirically observed in some markets.[1]

As in Lucia and Schwartz (2002), the process for the log-spot price is split into two components:

$$\ln(S_t) = f_t + Y_t$$

The first term, f_t, is the deterministic component that represents the seasonal behavior of the commodity price, which is modelled by a Fourier series:[2]

$$f_t = \sum_{n=0}^{\infty} \mathrm{Re}\left[A_n e^{inw_f t}\right], \quad w_f \in \mathbb{R}^+ \tag{4}$$

The second term, Y_t, follows a mean-reverting process which converges to $z(t)$, a time-dependent function that captures long-term variations:

$$dY_t = \kappa(z_t - Y_t)dt + \sigma dW_t \tag{5}$$

[1]See, for instance, Mu and Ye (2015) in the case of crude oil prices.

[2]This series is the basic mathematical tool of harmonic analysis and allows to decompose a function into an infinite sum of sinusoidal functions.

where κ and σ denote, respectively, the speed of mean-reversion and the diffusion coefficient, and W_t is a standard Wiener process. Moreover, we have:

$$z_t = \sum_{n=0}^{\infty} \text{Re}\left[B_n e^{inw_z t}\right], \quad w_z \in \mathbb{R}^+ \tag{6}$$

The coefficients of both Fourier series (see equations (4) and (6)), A_n and B_n, are complex numbers such that $A_n = A_{x,n} + iA_{y,n}$. $B_n = B_{x,n} + iB_{y,n}$. The two terms of these numbers denote, respectively, the amplitude and the phase of the Fourier representations for the functions f_t and z_t. Note that when $B_n = A_n = 0$ we obtain the Schwartz (1997) model and, if $B_n = 0$ and $f_t = \alpha + \beta D_t + \gamma \cos\left((t+\varphi)\frac{2\pi}{365}\right)$, we get the Lucia and Schwartz (2002) model.

Therefore, the Moreno *et al.* (2019) model generalizes both models and it incorporates seasonal and cyclical fluctuations as well as long-term fluctuations. These authors obtained closed-form expressions for the values of different derivatives and analyzed the empirical behavior of their model with data of heating oil, natural gas, and oil, and found that their model outperformed both particular cases.

Taking a constant risk market price λ, the process (5) can be written under the measure $\widetilde{P}$ as

$$dY_t = \mu_t dt + \sigma d\widetilde{W}_t$$

where $\mu_t = \kappa\left(\alpha + \tilde{z}_t - Y_t\right)$, $\alpha = B_0 - \dfrac{\sigma}{\kappa}\lambda$, $\tilde{z}_t$ is given by equation (6), and $\tilde{W}_t = W_t + \lambda t$.

The solution of this process is:

$$Y_s = e^{-\kappa(s-t)}Y_t + \left(1 - e^{-\kappa(s-t)}\right)\alpha + \sum_{n=1}^{\infty} \text{Re}\left[\frac{\kappa B_n}{\kappa + inw_z}\left(e^{inw_z s} - e^{-\kappa(s-t)+inw_z t}\right)\right] + \sigma \int_t^s e^{-\kappa(s-u)} d\tilde{W}_u$$

Hence, the conditional distribution of the log-spot price is Gaussian with expectation and variance

$$\begin{aligned}\widetilde{\mathbb{E}}_t\left[\ln\left(S_t\right)\right] =& f_T + e^{-\kappa(T-t)}\left(\ln\left(S_t\right) - f_t\right) + \left(1 - e^{-\kappa(T-t)}\right)\alpha \\ &+ \sum_{n=1}^{\infty} \mathrm{Re}\left[\frac{\kappa B_n}{\kappa + inw_z}\left(e^{inw_zT} - e^{-\kappa(T-t)+inw_zt}\right)\right] \\ \widetilde{\mathbb{V}}_t\left[\ln\left(S_t\right)\right] =& \frac{\sigma^2}{2\kappa}\left(1 - e^{-2\kappa(T-t)}\right)\end{aligned}$$

Using $F\left(S_t, t, T\right) = \widetilde{\mathbb{E}}_t\left[S_T\right]$ and properties of the lognormal distribution, the log-futures price is:

$$\begin{aligned}\ln\left(F\left(S_t, t, T\right)\right) =& f_T + e^{-\kappa(T-t)}\left(\ln\left(S_t\right) - f_t\right) + \left(1 - e^{-\kappa(T-t)}\right)\alpha + \frac{\sigma^2}{4k}\left(1 - e^{-2\kappa(T-t)}\right) \\ &+ \sum_{n=1}^{\infty} \mathrm{Re}\left[\frac{\kappa B_n}{\kappa + inw_z}\left(e^{inw_zT} - e^{-\kappa(T-t)+inw_zt}\right)\right]\end{aligned} \tag{7}$$

We can interpret the components of this expression as follows:

$$\begin{aligned}\ln\left(F\left(S_t, t, T\right)\right) =& \underbrace{f_T - e^{-\kappa(T-t)}f_t}_{\text{Short- and mid-term seasonal component}} + \underbrace{e^{-\kappa(T-t)}\ln\left(S_t\right)}_{\text{Spot price correction}} + \underbrace{\frac{\sigma^2}{4k}\left(1 - e^{-2\kappa(T-t)}\right)}_{\text{Volatility effect}} \\ &+ \underbrace{\left(1 - e^{-\kappa(T-t)}\right)\alpha + \sum_{n=1}^{\infty} Re\left[\frac{\kappa B_n}{\kappa + inw_z}\left(e^{inw_zT} - e^{-\kappa(T-t)+inw_zt}\right)\right]}_{\text{Long-term swing}}\end{aligned}$$

Alternatively, we can express the log-futures price as the sum of two terms:

$$\ln\left(F_t\left(S_t, t, T\right)\right) = M\left(S_t, t, T; \theta\right) + N(t, T; \theta)$$

where

$$M\left(S_t, t, T; \theta\right) = e^{-\kappa(T-t)}\ln\left(S_t\right) + \left(1 - e^{-\kappa(T-t)}\right)\alpha + \frac{\sigma^2}{4\kappa}\left(1 - e^{-2\kappa(T-t)}\right)$$

is the component that models the behavior of spot prices (used by Schwartz (1997) and Lucia and Schwartz (2002)) and

$$N(t, T; \theta) = f_T - e^{-\kappa(T-t)}f_t + \sum_{n=1}^{\infty} \mathrm{Re}\left[\frac{\kappa B_n}{\kappa + inw_z}\left(e^{inw_zT} - e^{-\kappa(T-t)+inw_zt}\right)\right]$$

is the component that includes the seasonal effect of prices and represents the novel aspect of this model when compared to the preceding models.

Moreno *et al.* (2019) proposed the following two particular cases:

- **Particular case 1**

  The simplest case is when we consider a single term of the Fourier series to represent the long-term swing. Thus, $N(t,T;\theta)$ results:

$$N(t,T;\theta) = Re\left[\frac{\kappa B}{\kappa + iw_z}\left(e^{iw_z T} - e^{-\kappa(T-t)+iw_z t}\right)\right]$$

- **Particular case 2**

  We add a term to the Fourier series, a single frequency for f_t that models the seasonal behavior. Therefore, $N(t,T;\theta)$ results:

$$N(t,T;\theta) = \mathrm{Re}\left[A\left(e^{iw_{f,1}T} - e^{-\kappa(T-t)+iw_{f,1}t}\right)\right] + \mathrm{Re}\left[\frac{\kappa B}{\kappa + iw_z}\left(e^{iw_z T} - e^{-\kappa(T-t)+iw_z t}\right)\right]$$

Del Campo and Moreno (2018) proposed a new particular case for agricultural commodities:

- **Particular case 3**

  These authors considered two frequencies for the short- and mid-term seasonal component and just one frequency for the long-term swing:

$$N(t,T;\theta) = \sum_{l=1,2} Re\left[A_l\left(e^{iw_{f,l}T} - e^{-\kappa(T-t)+iw_{f,l}t}\right)\right] + Re\left[\frac{\kappa B}{\kappa + iw_z}\left(e^{iw_z T} - e^{-\kappa(T-t)+iw_z t}\right)\right]$$

Balado-Alves and Moreno (2019) modeled the futures market for CO_2 emission allowances and proposed the following four specifications:

- **Particular case 4**

  This case assumes three frequencies in the Fourier series and one for the long-term swing, thus:

$$N(t,T;\theta) = \sum_{l=1,2,3} \mathrm{Re}\left[A_l\left(e^{iw_{f,l}T} - e^{-\kappa(T-t)+iw_{f,l}t}\right)\right] + Re\left[\frac{\kappa B}{\kappa + iw_z}\left(e^{iw_z T} - e^{-\kappa(T-t)+iw_z t}\right)\right]$$

- **Particular cases 5, 6, and 7**

  These cases consider no long-term swing and, respectively, 1, 2, 3 frequencies in the Fourier series. Thus, we have the following expressions:

$$N(t,T;\theta) = \mathrm{Re}\left[A\left(e^{iw_{f,1}T} - e^{-\kappa(T-t)+iw_{f,1}t}\right)\right]$$
$$N(t,T;\theta) = \sum_{l=1,2} \mathrm{Re}\left[A_l\left(e^{iw_{f,l}T} - e^{-\kappa(T-t)+iw_{f,l}t}\right)\right]$$
$$N(t,T;\theta) = \sum_{l=1,2,3} \mathrm{Re}\left[A_l\left(e^{iw_{f,l}T} - e^{-\kappa(T-t)+iw_{f,l}t}\right)\right]$$

4 Methodology and Data

In this section, we outline the estimation approach that will be used to evaluate the empirical performance of the previously mentioned pricing models. The assessment involves two main steps. Firstly, we employ an in-sample non-linearleast-squares regressiion to estimate the model parameters using historical data. Subsequently, we apply the Kalman filter technique to examine the predictive ability of the models by generating one-period-ahead forecasts. This methodology allows us to gain insights into the accuracy and effectiveness of each model in capturing the dynamics of electricity futures prices.

4.1 Data

For our analysis we are using daily closing prices for electricity futures taken from the European Energy Exchange (EEX). Specifically, we are using the spot and futures prices for the EEX-Phelix, which is the Physical Electricity Index for the German and Austrian Market. It is issued in Euros per Megawatt hour via the Phelix Base and Phelix Peak, which respectively represent an average daily price over all hours of the day for the base load and only from nine in the morning to eight in the evening for the peak load. In our case, we have based our analysis on the Phelix Base. The contract sizes for the electricity futures vary between maturities but generally move between 720 to 745 megawatt hours.

We obtained futures prices from Eikon Thomson Reuter's Datastream, where the roll-over at the end of a future's maturity is performed automatically. The complete sample includes data from July 1st, 2002, to June 1st, 2023. Each future in our time series is labelled starting with the abbreviation of the exchange we used, EEX, followed by a number that indicates the proximity to maturity. For instance, the EEX-5 indicates an electricity future that is fifth closest to maturity.

To be emphasized is the fact that our sample includes the highly volatile and disruptive time period during the Covid-19 pandemic and later, the ongoing war between Russia and Ukraine. Due to Russia's status as an important exporter of oil and natural gas, the electricity market experienced large shocks and price spikes immediately following the invasion of Ukraine. This is shown in Figures 1 to 6, that depict the different time series of spot and futures prices and of which the ones that are not mentioned here can be found in the corresponding Appendix.

Figure 1 depicts the electricity spot price movements up until July 2021. Apart from a single spike of up to 301.54€/Mwh in July 2006 the electricity spot price has remained relatively stable over these nineteen years.

Figure 2 extends the previous one by the last two years of our dataset and shows the spot price from June, 2002 until June 2023. It is very apparent that the Covid-19 pandemic and the Russian invasion have heavily disrupted the electricity market, with price spikes of up to 699.44€/Mwh in late August last year, close to 6 months after Russia's invasion of Ukraine.

Figure 3 puts into perspective how impactful these events were on the futures market as it depicts the spot and futures prices for all six maturities for our whole dataset. The extreme spikes of the EEX-6 and EEX-7 with prices of up to 1,423.82€/Mwh coincide with the shocks in the spot market. Contracts with longer time to maturity are more sensitive to economic shocks due to longer period of uncertainty. This also shows how stable the electricity market was up until two years ago and how unusual and unexpected the market movements of the last two years have been.

This feature of our dataset is also evident in Table 1 which shows the most relevant descriptive statistics of, first, the whole sample period, then, of the data up until July 1, 2021, and finally, the data from July 1, 2021, until June 1, 2023. The very high standard deviations and the large differences between the maxima and minima for the last sample period pay testament to the high volatility and uncertainty the electricity market has experienced over the last two years. In contrast, when excluding this highly volatile period, standard deviations are much lower and the mean is much closer to the minima and maxima, suggesting more stable price movements and less volatility.

Another peculiarity are the very high kurtosis values for the whole dataset and the relatively low kurtosis values for the timeframe until 01 July 2021, which suggest that including the last two years of data introduces extreme outliers into the dataset. The skewness is also affected by this, as, even though it is positive for both data subsets, it relatively gets much larger when including the last two years of data and therefore makes extremely high values more probable than extremely low ones.

Because of these characteristics of the last two years of our sample and since this thesis' objective is to determine which model performs best under more "regular" circumstances, we will be using the first part of our dataset, from July 1, 2002, until July 1, 2021, for our performance ranking of our chosen futures pricing models.

Table 2 provides an overview of the results of an Augmented Dickey-Fuller test on the first part of our data until 01 July 2021 and on the second part until 01 June 2023. As can be seen, for the series in differences, stationarity is evidenced, as all p-values are below our significance value of 0.05. For the series in levels however, the results differ between the two subsets of our dataset. While the time series of futures prices for second and fourth closest to maturity futures exhibits stationarity in the first dataset, for the second dataset, none of the futures time series exhibit stationarity in levels. This also shows that this subset of data contains time series with less stable means and covariances which make ordinary statistical analysis more difficult. However, as the results show, transforming the time series in both datasets via differenciating makes them stationary, even the more volatile second period.

4.2 In-sample analysis

Let P_{jt} denote the price at time t of the futures that matures at time j. For each model, the parameter vector $\beta = (\beta_1, \beta_2, \beta_3, \dots)$ will be estimated by non-linear least-squares regression. Then, we have the following optimization problem:

$$\min\left(SSR\left(\hat{\theta};\hat{\theta}_m\right)\right)=\sum_{t=1}^{T}\sum_{j=1}^{K}\left(P_{jt}-\beta'\eta_{j,t}\right)'W\left(P_{jt}-\beta'\eta_{j,t}\right)$$

where θ and θ_m are, respectively, the vectors of structural (common for all the models) and of cyclical parameters, K is the number of maturities, $W = I_K$, and $\beta'\eta_{j,t} = \sum_{i=1}^{12}\beta_i\eta_{ij,t}$.

The overall structure of commodity prices is consistent across futures contracts, it is important to note that specific seasonal and cyclical components can vary for each maturity. We can represent a non-linear (and time-dependent) function of the structural parameters as

$$P_t=\ln\left(F\left(S_t,t,T\right)\right)-e^{-\kappa(T-t)}\ln\left(S_t\right)=\sum_{i=1}^{12}\beta_i\eta_{ij,t}+u_t$$

Note that, by definition, this variable tends to zero when time goes by. Moreover, we have $\eta_{1t} = 1-e^{-\kappa(T-t)}$ and $\eta_{2t}=\dfrac{1}{4\kappa}\left(1-e^{-2\kappa(T-t)}\right)$, that are common for all the models.

For each model, we have the following parameters:

1. Schwartz (1997) only incorporates mean-reversion:

$$\beta_1=\tilde{\alpha}=\mu-\frac{\sigma^2}{2\kappa}-\frac{\sigma\lambda}{\kappa},\quad \beta_2=\sigma^2,\quad \beta_i=0,\ i=3,\ldots,12$$
$$\theta=(\tilde{\alpha},\kappa,\sigma)$$

2. Lucía and Schwartz (2002) adds a new factor to capture annual seasonality in prices:

$$\eta_{3t}=\cos\left((T+\varphi)\frac{2\pi}{260}\right)-e^{\kappa(T-t)}\cos\left((t+\varphi)\frac{2\pi}{260}\right)$$
$$\beta_1=\tilde{\alpha}=\mu-\frac{\sigma\lambda}{\kappa},\quad \beta_2=\sigma^2,\quad \beta_3=\gamma,\quad \beta_i=0,\ i=4,\ldots,12$$
$$\theta=(\tilde{\alpha},\kappa,\sigma,\gamma,\varphi)$$

In the next three models, we have $\beta_1 = \tilde{\alpha}$, $\beta_2 = \sigma^2$. Moreover, we have the following:

3. Moreno *et al.* (2019) [Particular case 1]:

$$\beta_3\eta_{3t} + \beta_4\eta_{4t} = \text{Re}\left[(B_x + iB_y)\frac{\kappa}{\kappa + iw_z}\left(e^{iw_zT} - e^{-\kappa(T-t)+iw_z\kappa t}\right)\right]$$
$$\beta_i = 0,\ i = 5, \ldots, 9$$
$$\theta = (\alpha, \kappa, \sigma, B_{x_1}, B_{y_1}, w_z)$$

4. Moreno *et al.* (2019) [Particular case 2]:

$$\beta_3\eta_{3t} + \beta_4\eta_{4t} = \text{Re}\left[(B_x + iB_y)\frac{\kappa}{\kappa + iw_z}\left(e^{iw_zT} - e^{-\kappa(T-t)+iw_z\kappa t}\right)\right]$$
$$\beta_5\eta_{5t} + \beta_6\eta_{6t} = \text{Re}\left[(A_{x,1} + iA_{y,1})\, e^{iw_{f,1}t}\right]$$
$$\beta_i = 0,\ i = 7, 8, 9$$
$$\theta = (\alpha, \kappa, \sigma, B_{x_1}, B_{y_1}, w_z)$$
$$\theta_m = \left(A^j_{x,1}, A^j_{y,1}, w^j_{f,1}\right),\ j = 1, 2, \ldots, K$$

5. Del Campo and Moreno (2018) [Particular case 3]:

$$\beta_3\eta_{3t} + \beta_4\eta_{4t} = \text{Re}\left[(B_x + iB_y)\frac{\kappa}{\kappa + iw_z}\left(e^{iw_zT} - e^{-\kappa(T-t)+iw_z\kappa t}\right)\right]$$
$$\sum_{i=5}^{8}\beta_i\eta_{it} = \sum_{l=1,2}\text{Re}\left[(A_{x,l} + iA_{y,l})\, e^{iw_{f,l}t}\right]$$
$$\beta_9 = 0$$

6. Balado-Alves and Moreno (2019) [Particular case 4]:

$$\beta_3\eta_{3,t} + \beta_4\eta_{4,t} = \text{Re}\left[(B_{x,1} + iB_{y,1})\frac{\kappa}{\kappa + iw_g}\left(e^{iw_gT} - e^{-\kappa(T-t)+iw_gt}\right)\right]$$
$$\sum_{i=5}^{10}\beta_i\eta_{i,t} = \sum_{l=1,2,3}\text{Re}\left[(A_{x,l} + iA_{y,l})\, e^{iw_{f,}t}\right]$$
$$\beta_{11} = \beta_{12} = 0$$

7. Balado-Alves and Moreno (2019) [Particular case 5]:

$$\beta_5\eta_{5,t} + \beta_6\eta_{6,t} = \mathrm{Re}\left[\left(A_{x,1} + iA_{y,1}\right)e^{iw_{f,1}t}\right]$$
$$\beta_i = 0,\ i = 3, 4, 7, \ldots, 12$$

8. Balado-Alves and Moreno (2019) [Particular case 6]:

$$\sum_{i=5}^{8}\beta_i\eta_{i,t} = \sum_{l=1,2}\mathrm{Re}\left[\left(A_{x,l} + iA_{y,l}\right)e^{iw_{f,l}t}\right]$$
$$\beta_i = 0,\ i = 3, 4, 9, 10, 11, 12$$

9. Balado-Alves and Moreno (2019) [Particular case 7]:

$$\sum_{i=5}^{10}\beta_i\eta_{i,t} = \sum_{l=1,2,3}\mathrm{Re}\left[\left(A_{x,l} + iA_{y,l}\right)e^{iw_{f,it}}\right]$$
$$\beta_i = 0,\ i = 3, 4, 11, 12$$

4.3 Out-of-sample analysis

As mentioned before, we will apply the Kalman filter to perform the forecasting analysis of the models. First, we briefly describe the technique.

4.3.1 Kalman filter

The Kalman filter, introduced by Kalman (1960), is widely used tool for estimation, as it is an algorithm that can estimate both observable and unobservable parameters with great accuracy and in real-time. The limitation of this model however, is that it assumes that no observable variable can affect unobservable states or variables. As future spot prices are unobservable, we perform the predictive analysis by applying this methodology that is adequate for latent variables and has been chosen against other alternatives due to its quickness and ease of implementation, see Galli and Lautier (2004). This technique has been widely used in studies as Schwartz (1997), Manoliu and Tompaidis (1999), Schwartz and Smith (2000), or Moreno *et al.* (2019), among others.

This filter is a recursive algorithm that, as new information arrives, sequentially updates the linear projection of a system of variables on the set of available information. This allows to numerically evaluate a likelihood function, generating estimates of the latent unobservable variables. For this purpose, we must represent the corresponding model in the state-space formulation. Following Hamilton (1994), we get

$$\begin{aligned} \xi_{t+1} &= F\xi_t + \nu_{t+1} && \text{(State equation)} \\ y_t &= A'x_t + H'\xi_t + \omega_t && \text{(Observation equation)} \end{aligned} \tag{8}$$

where ξ_t is a vector of unobservable variables while y_t and x_t include observable variables. Specifically, x_t contains exogenous variables, i.e., it does not contain information about ξ_{t+s} or ω_{t+s} that is not already contained in the lags of the variable y_t.

4.3.2 Kalman filter specification

The daily closing prices of futures contracts are determined by averaging the prices observed during the final minutes of the trading session. As a result, futures prices are generally more smooth when compared to the spot prices. It is important to consider that spot prices incorporate some level of noise. To address this, we employ the Kalman filter technique to estimate the spot price, S_t, at a given time t, and use this estimation to predict the spot price S_{t+1} for the subsequent period.

In our case, we obtain the following equations:

$$\begin{aligned} \ln\left(S_{t+1}\right) &= G + F\ln\left(S_t\right) + \nu_{t+1} && \text{(State equation)} \\ \ln\left(F\left(S_t, t, T\right)\right) &= A'x_t + e^{-\kappa(T-t)}\ln\left(S_t\right) + \omega_t && \text{(Observation equation)} \end{aligned}$$

where $\mathbb{E}\left(v_t v_t'\right) = Q$ and $\mathbb{E}\left(\omega_t \omega_t'\right) = R$, being Q and R constants.

The matrices included in the observation equation are

$$A' = \begin{pmatrix} \alpha & \frac{\sigma^2}{4\kappa} & \gamma & \gamma & B_x & B_y & A_x & A_y & A_{x,2} & A_{y,2} & A_{x,3} & A_{y,3} \end{pmatrix}$$

$$
x_t = \begin{pmatrix}
1 - e^{-\kappa(T-t)} \\
1 - e^{-2\kappa(T-t)} \\
\cos\left((T+\varphi)\frac{2\pi}{260}\right) \\
-e^{-\kappa(T-t)}\cos\left((t+\varphi)\frac{2\pi}{260}\right) \\
\kappa\cos(w_z T) + w_z\sin(w_z T) - e^{-\kappa(T-t)}\left[\kappa\cos(w_z t) + w_z\sin(w_z t)\right] \\
-\kappa\sin(w_z T) + w_z\cos(w_z T) - e^{-\kappa(T-t)}\left[w_z\cos(w_z t) - \kappa\sin(w_z t)\right] \\
\cos(\omega_f T) - e^{-\kappa(T-t)}\cos(\omega_f t) \\
e^{-\kappa(T-t)}\sin(\omega_f t) - \sin(\omega_f T) \\
\cos(\omega_{f_2} T) - e^{-\kappa(T-t)}\cos(\omega_{f_2} t) \\
e^{-\kappa(T-t)}\sin(\omega_{f_2} t) - \sin(\omega_{f_2} T) \\
\cos(\omega_{f_3} T) - e^{-\kappa(T-t)}\cos(\omega_{f_3} t) \\
e^{-\kappa(T-t)}\sin(\omega_{f_3} t) - \sin(\omega_{f_3} T)
\end{pmatrix}
$$

Thus, each model will be related to the following rows of the matrices A' and x_t:

- Model 1: Rows 1 and 2.
- Model 2: Rows 1 to 4.
- Model 3: Rows 1, 2, 5 and 6.
- Model 4: Rows 1, 2, and 5 to 8.
- Model 5: Rows 1, 2, and 5 to 10.
- Model 6: Rows 1, 2, and 5 to 12.
- Model 7: Rows 1, 2, 7, and 8.
- Model 8: Rows 1, 2, and 7 to 10.
- Model 9: Rows 1, 2, and 7 to 12.

For each model, we will compute predictions for each quarter of 2020. This particular year was chosen because it represents the most recent period before the disruptions caused by both the Covid-19 pandemic and the Russian invasion of Ukraine. As mentioned before, we first need to determine the initial state and parameters of the model.

To do so, we take the previous year worth of data (01-01-2019 to 01-01-2020) to calibrate the model, exception are frequency parameters ω of the Fourier series, which we determine based on the full sample. Once the model is calibrated, we can obtain daily one-day-ahead prices for each quarter. The Kalman filter operates recursively, updating the state estimates and making predictions as new data becomes available. Therefore, it provides more accurate and refined estimates of the system's state over time. After predicting a quarter, we re-calibrate the model with new three month's worth of data and get new predictions, which we ultimately compare against the real observed values.

5 Empirical Analysis

5.1 In-sample Analysis

We now present and discuss the in-sample results obtained for electricity futures. We consider four measures of goodness-of-fit:

- Residual sum of squares (RSS): $\sum_{t=1}^{T} \min SCR\left(\hat{\theta}_t\right) = \sum_{t=1}^{T} \hat{u}_t^2$.
- Residual Standard Error (RSE): $\left(\frac{1}{n}\sum_{t=1}^{n} \hat{u}_t\right)^{1/2}$.
- Mean absolute error (MAE): $\frac{1}{n}\sum_{t=1}^{n} |\hat{u}_t|$.
- Akaike information criterion (AIC): $2k - 2\ln(\hat{L})$ or, alternatively, $AIC = 2k + n \cdot \ln\left(\frac{RSS}{n}\right)$.

The first three measures provide an assessment of the absolute goodness-of-fit of a model, evaluating how well it fits the observed data. On the other hand, the Akaike Information Criterion (AIC) is a relative measure that takes into account the balance between the model's empirical fit and its simplicity. It penalizes the inclusion of extra parameters, allowing us to compare different models. Additionally, the Residual Squared Error (RSE) is commonly used as an approximation of the standard deviation of the Residual Sum of Squares (RSS). It serves as a useful indicator of the model's predictive accuracy and the spread of the residuals.

Table 3 shows the estimates of our parameters for each model, as well as their measures of goodness-of-fit and Table 4 shows the relative improvement of each model compared to the previous one or the benchmark models, split into each contract. Table 5 shows the in-sample performance ranked by both Residual Sum of Squares and the Akaike-Information Criterion. The main results are as follows:

- Model 2: This model introduces seasonality into the analysis by incorporating a sinusoidal function. Upon comparing it to the benchmark model (Model 1), the improvement in terms of error reduction is relatively significant. One possible reason for it not being much higher is that the sinusoidal function used in Model 2 assumes an annual seasonality, which might not accurately capture the true seasonal effects present in the data.

- Model 3: This model introduces a Fourier series which aims to capture long-term swings in the equilibrium level that prices revert to. In terms of RSS reduction compared to the previous model, the improvement is significant. However, due to the number of degrees of freedom increasing by one compared to the previous model, the Akaike criterion slightly increased. It should be noted that, for the first two futures closest to maturity, a slight worsening has been observed, -5.32% and -6.09% respectively. This may be a signal that empirically there is no long-term mean-reversion level for futures with such a nearness to maturity. In addition, it is observed that improvement occurs gradually as we increase the proximity to maturity, up until EEX-7, when the improvement is still existent and significant, but lower than improvement observed for EEX-6.

- Model 4: This model adds a new Fourier series to capture short- and mid-term seasonal behaviours and it does reduce the RSS significantly with respect to the previous model. It is noticeable that largest improvement had been achieved for the two futures with the nearest maturities, 49.33% for EEX-2 and 67.71% for EEX-3. This indicates that adding one Fourier series captures a seasonality period of 1 year the best in near-to-maturity futures. The long-term swing period estimation stays the same as for the previous model, 13.6 years.

- Model 5: This model adds an extra Fourier series to capture short- and mid-term seasonal behaviours and it generates a significant improvement in RSS when compared to the previous model. This suggests that there is a presence of seasonality in the electricity price series, but this time the largest improvement has been achieved for EEX-5 and EEX-6, of 29.88% and 34.89% respectively. In addition, this improvement is supported by the AIC which increased in value, as it penalizes the introduction of more degrees of freedom.

- Model 6: This model adds a third frequency to capture short- and mid-term seasonal behaviours, but the improvement over the previous model is not significant.

- Model 7: This model uses one frequency in the Fourier series to capture short- and mid-term seasonal behaviours and no long-term swing. When comparing this model to Model 4 (LTS + one short- and mid-term seasonality term), the results in RSS significantly worsen for all the maturities, by -46.37% on average across all six maturities. This suggests the presence of oscillations in the long-term reversion value and the need to account for it.

- Model 8: This model uses two frequencies in the Fourier series to capture short- and mid-term seasonal behaviours and no long-term swing. As expected, this model outperforms Model 7 significantly, however in under performs Model 5 (LTS + 2 Fourier series for mid- and short-term seasonalities). Eliminating the LTS again proves to worsen the results.

- Model 9: This model adds a new frequency for short term seasonality and significantly redues RSS when comparing to the previous model. However, when comparing it with Model 6 (LTS + 3 Fourier series for mid- and short-term seasonalities), there is a slight worsening in the error reduction. This might be due to the possibility that including three Fourier series is enough to capture both long- and medium-/short-term seasonalities.

Therefore, we conclude that Model 6 outperforms all the rest, demonstrating that Moreno *et al.* (2019) surpasses the benchmark models.

5.2 Out-of-sample estimation

In this section, we aim to determine which model has the best predictive ability. To do so, we will analyse the observed errors between the values estimated by each model and their real values. The chosen out-of-sample period were the four quarters of 2020, which left us with the most data for the calibration of the parameters of the models whilst excluding the highly disruptive period in 2021 and beyond. Table 6 shows the ranking of the models by cumulative squared errors, whereas Table 7 gives a detailed overview of the generated squared errors for each model categorized by quarters and by closeness to maturity.

For the out-of-sample analysis, Model 2 has proved to be the best, with a cumulative error value for all contracts and quarters of 14.9779, followed by another benchmark model, Model 1 with a cumulative error of 16.9664. This proves that the models with the best predictive ability are not the ones with the best in-sample fit. This is also supported by the fact that Model 6, the model with the best in-sample fit, turned out to have the least predictive ability with a total error of 37.48. Model 5, with the second highest in-sample fit is also relatively unsuited for out-of-sample prediction, occupying the third-to-last place with a cumulative error of 27.03. This indicates that some overfitting might be occurring, or a greater range of futures contracts need to be considered for the calibration of the models.

A trend that can be observed in the out-of-sample results is that they seem to get worse if a model combines a long-term swing with increasing seasonality terms, as the cumulative errors for Models 4, 5 and 6 increase with each added Fourier term. However, this trend is reversed for Models 7, 8 and 9, whose errors decrease with each added seasonality term. This would suggest that, without the long-term swing, adding seasonalities captures the out-of-sample values better, whereas, with the long-term swing, adding more than one seasonality term leads to overfitting. This is also supported by the fact that Model 4, having a long-term swing and only one seasonality, is the third best model in terms of predictive capability. This makes it the best one for out-of-sample forecasting out of all the models that incorporate seasonality with Fourier terms.

This evidence suggests that models using Fourier series are not superior to the rest in a out-of-sample analysis, which can be seen in Table 8, which shows chosen models ranked by their out-of-sample forecasting errors. These results are in line with those found in papers that perform this study on other asset classes, such as Balado-Alves and Moreno (2019) and Göransson and Moreno (2022), who analyzed futures on, respectively, CO2 emissions and precious metals.

6 Conclusions

With renewable energy production gaining significant attention in projects that attempt to combat the climate change, it is becoming obligatory for companies to maintain their reputation and engage in usage of renewable energy. Due to dependency on weather conditions and non-storability issues of renewable energy, markets of electricity derivatives are turning into an essential part of any risk management strategy. Among these tools, electricity futures play an important role, as they allow for a reduction in exposure to sudden electricity price shocks by locking in a certain price ahead of time. Modelling and forecasting electricity spot and futures prices may therefore benefit companies investing in renewable energy projects by improving their ability to plan ahead and better manage their price risk exposure.

Even though we have established that electricity can not be directly compared to other commodities such as precious metals or agricultural products, it does exhibit a certain price behaviour that can be found in all commodities. Two of these price characteristics are mean-reversion and seasonality, which the models studied in this thesis all account for in different ways. Specifically, we studied the in- and out-of-sample performance of two benchmark models and seven particular cases of a third model, proposed in Moreno, Novales and Platania (2019). This model allows for a high flexibility as these authors propose very general types of seasonalities and, then, we can determine whether our chosen extensions may improve the (in-sample) performance of previous models as well as their forecasting capability.

We have used data from the European Electricity Exchange of electricity spot and futures prices in order to compared each model's goodness-of-fit and we have ranked all the analyzed models in terms of their in- and out-of-sample performance. The main qualitative conclusion of the in-sample analysis is that models that account for seasonality modelled by using Fourier series are clearly superior to the benchmark models.

On the other hand, for the out-of-sample forecasting analysis, results were less conclusive. Firstly, we should note that out-of-sample forecasting is always expected to yield larger errors, which could be due to overfitting. The model with the least errors when forecasting our chosen out-of-sample period of 2020 was the model proposed by Lucia and Schwartz (2002), whereas the model with the best in-sample performance, Model 6 (accounting for three terms of short- and medium seasonality), ended up in last place. This suggests that accounting for different types of seasonality with Fourier series and adding multiple terms to it in order to capture more seasonality frequencies is useful for the in-sample analysis, but less effective for the out-of-sample forecasting. However, iIt should be mentioned that Model 4 (incorporating a long-term swing and one Fourier series term) placed third in the ranking for the out-of-sample performance, perhaps suggesting that this variety of the model can be useful for electricity price forecasting. However, further research into testing this particular model would be required to thoroughly assess its potential for practical application.

As for further lines of research, we can suggest analysing the empirical behaviour of a two-factor model that incorporates convenience yield as a second factor, as proposed in Bacaicoa, Moreno, and Platania (2014). Moreover, we can also consider incorporating non-constant or seasonal volatility, to better capture noise in the data. In our research, we have used data from EEX (German and Austrian electricity market), however there are few more markets that can also be explored and potentially offer different insights, such as Nord Pool (the Nordic energy market), the NEM (National Electricity Market) in Australia, or the PJM (Pennsylvania-New Jersey-Maryland, the U.S energy market).

Electricity will always remain directly linked to the global mission against climate change. Knowing how its price moves over time and being able to adjust their risk strategy accordingly will be invaluable for companies that invest in renewable energy. By showcasing possible ways to model electricity prices and account for its intricacies, we attempt to do our part towards achieving a more sustainable future.

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Appendix of Tables

	Mean	Std. Dev.	Min.	Max.	Skewness	Kurtosis
EEX-spot	57.3911	59.5674	1.52	699.44	4.8297	29.2763
EEX-2	57.8751	62.5551	17.14	674.75	4.3165	21.5623
EEX-3	60.5958	71.9207	19.19	794.03	4.7262	26.6039
EEX-4	61.9976	78.9791	20.50	1080.68	5.3681	36.0526
EEX-5	62.6775	83.9267	20.80	1151.98	5.8430	42.1090
EEX-6	62.5486	83.7787	20.81	1423.82	6.3382	53.2805
EEX-7	61.7308	76.8455	20.35	1399.87	5.9778	51.3069
	Mean	Std. Dev.	Min.	Max.	Skewness	Kurtosis
EEX-spot	43.3465	16.7334	1.52	301.54	2.2861	17.9002
EEX-2	41.6834	12.9432	17.14	98.41	0.9992	1.0859
EEX-3	42.4521	13.2395	19.19	96.76	0.9621	0.9224
EEX-4	42.8231	13.3368	20.50	98.23	1.0795	1.5284
EEX-5	43.0386	13.3675	20.80	101.94	1.1469	1.9662
EEX-6	43.3062	13.4833	20.81	101.00	1.1123	1.9572
EEX-7	43.4476	13.3432	20.35	102.75	1.0258	1.6993
	Mean	Std. Dev.	Min.	Max.	Skewness	Kurtosis
EEX-spot	196.47	120.9146	12.13	699.44	2.2861	17.9002
EEX-2	218.20	112.6927	74.26	674.75	0.9992	1.0859
EEX-3	240.24	138.2937	77.88	794.03	0.9621	0.9224
EEX-4	251.84	162.9985	81.76	1080.68	1.0795	1.5284
EEX-5	257.13	182.7749	84.93	1151.98	1.1469	1.9662
EEX-6	253.07	186.4792	81.30	1423.82	1.1123	1.9572
EEX-7	242.85	162.8098	85.22	1399.87	1.0258	1.6993

Table 1: **Key statistics for electricity spot and futures prices.** This Table reports key statistics (mean, standard deviation, minimum, maximum, skewness, and kurtosis) for 3 datasets. Starting from the top, the sample periods are: 01 July, 2002 to 01 June 2023, 01 July, 2002 to 01 July, 2021 and 01 July, 2021 to 01 June, 2023. Each futures is indicated by the first three letters of the commodity followed by a number that indicates the order it occupies with respect to maturity.

	Series in levels	Series in differences		Series in levels	Series in differences
	$t-$ stat. ($p-$value)	$t-$ stat. ($p-$value)		$t-$ stat. ($p-$value)	$t-$ stat. ($p-$value)
EEX-Spot	-4.7739 (0.0006)	-19.3536 (0.000)	EEX-Spot	-3.5332 (0.0072)	-16.0079 (0.000)
EEX-2	-3.5082 (0.0078)	-12.1060 (0.000)	EEX-2	-2.1413 (0.2282)	-21.0158 (0.000)
EEX-3	-2.7729 (0.0623)	-68.7527 (0.000)	EEX-3	-1.9690 (0.3003)	-11.7609 (0.000)
EEX-4	-2.9238 (0.0426)	-11.1260 (0.000)	EEX-4	-1.8780 (0.3424)	-8.5681 (0.000)
EEX-5	-1.9166 (0.3243)	-32.9246 (0.000)	EEX-5	-1.7464 (0.4074)	-8.8052 (0.000)
EEX-6	-2.3283 (0.1630)	-69.4877 (0.000)	EEX-6	-1.8910 (0.3363)	-8.8168 (0.000)
EEX-7	-1.8041 (0.3784)	-50.0384 (0.000)	EEX-7	-1.9875 (0.2921)	-12.2683 (0.000)

Table 2: **Augmented Dickey Fuller Test.** This Table reports $t-$statistics and $p-$values (in parenthesis) for electricity spot and futures prices for series in levels and in differences. The sample period for the left side is from 01 July, 2001 to 01 July, 2021. For the right side the sample period is from 01 July, 2021 until 01 June, 2023. Each futures is indicated by the first three letters of the commodity followed by a number that indicates the order it occupies with respect to maturity.

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
Structural parameters									
$\bar{\alpha}$	3.7293 (0.0022)	3.7295 (0.0020)	3.7366 (0.0011)	3.7432 (0.0010)	3.7469 (0.0009)	3.7469 (0.0009)	3.7345 (0.0021)	3.7328 (0.0019)	3.7311 (0.0016)
σ^2	1.28E-13	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)
κ	1.4925 (0.0169)	1.5055 (0.0117)	2.7830 (0.0268)	2.9671 (0.2476)	3.6312 (0.0314)	4.3987 (0.0411)	1.6401 (0.0151)	1.7729 (0.0184)	2.0043 (0.0257)
γ	-	-0.0823 (0.0011)	-	-	-	-	-	-	-
ϕ	-	1.8288 (0.0022)	-	-	-	-	-	-	-
B_{x_1}	-	-	-0.3235 (0.0023)	-0.3325 (0.0040)	-0.3137 (0.0015)	-0.3062 (0.0017)	-	-	-
B_{y_1}	-	-	0.0267 (0.0034)	0.0293 (0.0069)	0.0269 (0.0025)	0.0328 (0.0033)	-	-	-
$\omega_{z,1}$	-	-	2π0.0732(0.0010)	2π0.0735(0.0030)	2π0.0727(0.0007)	2π0.0732(0.0008)	-	-	-
Period	-	-	13.7 years	13.6 years	13.7 years	13.7 years	-	-	-
Seasonal parameters for EEX-2									
$A_{x,1}$	-	-	-	-0.0674 (0.0239)	-0.0781 (0.0041)	-0.0886 (0.0044)	0.0324 (0.0231)	-0.3417 (0.0130)	-0.3282 (0.0123)
$A_{y,1}$	-	-	-	0.0617 (0.0249)	0.0584 (0.0046)	0.0564 (0.0065)	0.0901 (0.0119)	0.0739 (0.0350)	0.0547 (0.0196)
$\omega_{f,1}$	-	-	-	2π0.9970(0.0350)	2π0.9960(0.0052)	2π0.9956(0.0064)	2π0.2896(0.0198)	2π0.0745(0.0083)	2π0.0740(0.0051)
Period	-	-	-	1 years	1 years	1 years	3.4 year	1 years	1 years
$A_{x,2}$	-	-	-	-	0.0191 (0.0112)	0.0364 (0.0091)	-	-0.0502 (0.0060)	-0.0557 (0.0052)
$A_{y,2}$	-	-	-	-	0.1637 (0.0052)	0.1451 (0.0065)	-	0.0571 (0.0053)	0.0561 (0.0051)
$\omega_{f,2}$	-	-	-	-	2π0.1709(0.0059)	2π0.1728(0.0081)	-	2π0.9968(0.0092)	2π0.9957(0.0080)
Period	-	-	-	-	5.9 years	5.8 years	-	1 years	1 years
$A_{x,3}$	-	-	-	-	-	0.0358 (0.0113)	-	-	0.0932 (0.0180)
$A_{y,3}$	-	-	-	-	-	0.1038 (0.0089)	-	-	0.1599 (0.0132)
$\omega_{f,3}$	-	-	-	-	-	2π0.2930(0.0085)	-	-	2π0.1735(0.0091)
Period	-	-	-	-	-	3.4 years	-	-	5.8 years
Seasonal parameters for EEX-3									
$A_{x,1}$	-	-	-	-0.0630 (0.0102)	-0.0707 (0.0034)	-0.0796 (0.0032)	0.0496 (0.0103)	0.0490 (0.0103)	0.0445 (0.0098)
$A_{y,1}$	-	-	-	0.0632 (0.0065)	0.0649 (0.0035)	0.0661 (0.0038)	0.0721 (0.0090)	0.0778 (0.0086)	0.0884 (0.0071)
$\omega_{f,1}$	-	-	-	2π0.9970(0.0119)	2π0.9967(0.0043)	2π0.9962(0.0041)	2π0.2918(0.0123)	2π0.2916(0.0115)	2π0.2935(0.0091)
Period	-	-	-	1 years	1 years	1 years	3.4 year	3.4 years	3.4 years
$A_{x,2}$	-	-	-	-	0.0183 (0.0076)	0.0352 (0.0075)	-	0.4380 (0.1154)	-0.0478 (0.0041)
$A_{y,2}$	-	-	-	-	0.1478 (0.0038)	0.1338 (0.0041)	-	0.1701 (0.0059)	0.9956 (0.0078)
$\omega_{f,2}$	-	-	-	-	2π0.1682(0.0045)	2π0.1701(0.0059)	-	2π0.9956(0.0078)	2π0.9968(0.0069)
Period	-	-	-	-	5.9 years	5.8 years	-	1 years	1 years
$A_{x,3}$	-	-	-	-	-	0.0407 (0.0080)	-	-	0.0629 (0.0113)
$A_{y,3}$	-	-	-	-	-	0.0946 (0.0045)	-	-	0.1943 (0.0071)
$\omega_{f,3}$	-	-	-	-	-	2π0.2926(0.0065)	-	-	2π0.1608(0.0050)
-									
Period	-	-	-	-	-	3.4 years	-	-	6.2 years
Seasonal parameters for EEX-4									
$A_{x,1}$	-	-	-	-0.0580 (0.0093)	-0.0618 (0.0036)	-0.0676 (0.0037)	0.0640 (0.0072)	0.0630 (0.0067)	0.0608 (0.0067)
$A_{y,1}$	-	-	-	0.0756 (0.0097)	0.0816 (0.0030)	0.0857 (0.0035)	0.0516 (0.0088)	0.0561 (0.0068)	0.0684 (0.0065)
$\omega_{f,1}$	-	-	-	2π0.9977(0.0136)	2π0.9977(0.0040)	2π0.9972(0.0040)	2π0.2927(0.0110)	2π0.2926(0.0094)	2π0.2940(0.0078)
Period	-	-	-	1 years	6 years	1 years	3.4 years	3.4 years	3.4 years
$A_{x,2}$	-	-	-	-	0.0233 (0.0064)	0.0352 (0.0075)	-	-0.0407 (0.0044)	-0.0478 (0.0041)
$A_{y,2}$	-	-	-	-	0.1351 (0.0032)	0.1338 (0.0041)	-	0.0589 (0.0033)	0.0539 (0.0038)
$\omega_{f,2}$	-	-	-	-	2π0.1666(0.0042)	2π0.1701(0.0059)	-	2π0.9994(0.0073)	2π0.9968(0.0069)
Period	-	-	-	-	6 years	5.9 years	-	1 years	1 years
$A_{x,3}$	-	-	-	-	-	0.0589 (0.0052)	-	-	0.0661 (0.0084)
$A_{y,3}$	-	-	-	-	-	0.0807 (0.0041)	-	-	0.1817 (0.0057)
$\omega_{f,3}$	-	-	-	-	-	2π0.2929(0.0048)	-	-	2π0.1588(0.0039)
Period	-	-	-	-	-	3.4 years	-	-	6.2 years
-									

Table 3: **In-sample estimation for electricity futures.**. This Table reports estimates of structural and seasonal parameters (with $t-$statistics in parenthesis) and four measures of goodness-of-fit for electricity futures. The in-sample period is from 01 July, 2002 to 01 July, 2021. Each futures is indicated by the first three letters of the commodity followed by a number that indicates the order it occupies with respect to maturity.

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
Period	-	-	-	-	-	3.4 years	-	-	6.2 years
-									
Seasonal parameters for EEX-5									
$A_{x,1}$	-	-	-	-0.0498 (0.0043)	-0.0469 (0.0042)	-0.0474 (0.0041)	0.0726 (0.0052)	0.0724 (0.0050)	0.0717 (0.0050)
$A_{y,1}$	-	-	-	0.0927 (0.0047)	0.1010 (0.0027)	0.1062 (0.0026)	0.0384 (0.0067)	0.0425 (0.0065)	0.0543 (0.0059)
$\omega_{f,1}$	-	-	-	2π0.9980(0.0043)	2π0.9984(0.0038)	2π0.9980(0.0036)	2π0.2925(0.0082)	2π0.2926(0.0079)	2π0.2939(0.0066)
Period	-	-	-	1 years	6 years	1 years	3.4 years	3.4 years	3.4 years
$A_{x,2}$	-	-	-	-	0.0336 (0.0055)	0.0481 (0.0093)	-	-0.0379 (0.0048)	-0.0391 (0.0046)
$A_{y,2}$	-	-	-	-	0.1281 (0.0031)	0.1147 (0.0048)	-	0.0744 (0.0041)	0.0791 (0.0031)
$\omega_{f,2}$	-	-	-	-	2π0.1662(0.0039)	2π0.1679(0.0083)	-	2π1.0014(0.0062)	2π1.0008(0.0055)
Period	-	-	-	-	6 years	5.9 years	-	1 year	1 year
$A_{x,3}$	-	-	-	-	-	0.0704 (0.0052)	-	-	0.0729 (0.0067)
$A_{y,3}$	-	-	-	-	-	0.2922 (0.0056)	-	-	0.1710 (0.0049)
$\omega_{f,3}$	-	-	-	-	-	2π0.4713(0.0064)	-	-	2π0.1581(0.0034)
Period	-	-	-	-	-	3.4 years	-	-	6.3 years
Seasonal parameters for EEX-6									
$A_{x,1}$	-	-	-	-0.0329 (0.0059)	-0.0224 (0.0048)	-0.0176 (0.0048)	0.0813 (0.0041)	0.0827 (0.0040)	0.0831 (0.0041)
$A_{y,1}$	-	-	-	0.1117 (0.0028)	0.1176 (0.0024)	0.1199 (0.0022)	0.0241 (0.0064)	0.0269 (0.0063)	0.0393 (0.0059)
$\omega_{f,1}$	-	-	-	2π0.9977(0.0057)	2π0.9984(0.0037)	2π0.9983(0.0038)	2π0.2928(0.0072)	2π0.2931(0.0067)	2π0.2941(0.0058)
Period	-	-	-	1 year	1 year	1 year	3.4 years	3.4 years	1 year
3.4 years									
$A_{x,2}$	-	-	-	-	0.0495 (0.0051)	0.0632 (0.0086)	-	-0.0361 (0.0057)	-0.0342 (0.0054)
$A_{y,2}$	-	-	-	-	0.1235 (0.0032)	0.1085 (0.0057)	-	0.0954 (0.0038)	0.0990 (0.0031)
$\omega_{f,2}$	-	-	-	-	2π0.1664 (0.0037)	2π0.1679(0.0082)	-	2π1.0011(0.0053)	2π1.0003(0.0049)
Period	-	-	-	-	6 years	5.9 years	-	1 year	1 year
$A_{x,3}$	-	-	-	-	-	0.0846 (0.0043)	-	-	0.0847 (0.0066)
$A_{y,3}$	-	-	-	-	-	0.0568 (0.0054)	-	-	0.1636 (0.0048)
$\omega_{f,3}$	-	-	-	-	-	2π0.2921(0.0053)	-	-	2π0.1580(0.0034)
-									
Period	-	-	-	-	-	3.4 years	-	-	6.3 years
-									
Seasonal parameters for EEX-7									
$A_{x,1}$	-	-	-	-0.0117 (0.0195)	0.0009 (0.0050)	0.0067 (0.0049)	0.0879 (0.0038)	0.0892 (0.0038)	0.0936 (0.0035)
$A_{y,1}$	-	-	-	0.1244 (0.0083)	0.1238 (0.0024)	0.1214 (0.0022)	0.0138 (0.0063)	0.0274 (0.0067)	0.0269 (0.0055)
$\omega_{f,1}$	-	-	-	2π0.9965(0.0128)	2π0.9973(0.0036)	2π0.9974(0.0036)	2π0.2929(0.0063)	2π0.2932(0.0059)	2π0.2941(0.0049)
Period	1 year	1 year	1 year	3.4 years	3.4 years	1 year	3.4 years	3.4 years	1 year
3.4 years									
$A_{x,2}$	-	-	-	-	0.0625 (0.0047)	0.0750 (0.0059)	-	0.1026 (0.0065)	0.0953 (0.0060)
$A_{y,2}$	-	-	-	-	0.1089 (0.0035)	0.0923 (0.0052)	-	0.1452 (0.0055)	0.1487 (0.0048)
$\omega_{f,2}$	-	-	-	-	2π0.1666 (0.0039)	2π0.1680(0.0063)	-	2π0.1574(0.0036)	2π0.1575(0.0032)
Period	-	-	-	-	6 years	5.9 years	-	6.3 years	6.3 years
$A_{x,3}$	-	-	-	-	-	0.0962 (0.0038)	-	-	-0.0260 (0.0063)
$A_{y,3}$	-	-	-	-	-	0.0463 (0.0061)	-	-	0.1233 (0.0033)
$\omega_{f,3}$	-	-	-	-	-	2π0.2917(0.0050)	-	-	2π0.9986(0.0045)
Period	-	-	-	-	-	3.4 years	-	-	1 year
Measures of goodness-of-fit									
$\sum_t \hat{u}_t^2$	1.68E+03	1.52E+03	1.09E+03	8.88E+02	7.25E+02	616.6918	1.62E+03	1.42E+03	1.22E+03
$\left(\frac{1}{n}\sum_{t=1}^{n}\hat{u}_t\right)^{1/2}$	0.23747	0.2258	0.1911	0.1727	0.1561	0.144	0.2332	0.2186	0.2027
$\frac{1}{n}\sum_{t=1}^{n}\lvert\hat{u}_t\rvert$	0.1759	0.1675	0.143	0.1308	0.1308	0.102	0.1745	0.1634	0.1529
AIC	-8.56E+04	-8.86E+04	-9.85E+04	-1.03E+05	-1.10E+05	-1.15E+05	-8.66E+04	-9.04E+04	-9.49E+04

	EEX-2	EEX-3	EEX-4	EEX-5	EEX-6	EEX-7
Model 1						
RSS	224.51	246.95	247.98	255.54	275.03	365.04
Model 2						
RSS	208.71	227.30	222.50	221.45	231.84	314.12
Improvement over Model 1	7.57%	8.64%	11.45%	15.39%	18.63%	16.21%
Seasonality (years)	1	1	1	1	1	1
Model 3						
RSS	220.45	242.03	160.72	153.53	153.21	223.43
Improvement over Model 2	-5.32%	-6.09%	38.44%	44.24%	51.32%	40.59%
LTS (years)	13.8	13.7	13.7	13.6	13.6	13.6
Model 4						
RSS	147.62	144.31	126.13	114.18	110.96	180.80
Improvement over Model 3	49.33%	67.71%	27.43%	34.46%	38.07%	23.58%
Seasonality (years)	1	1	1	1	1	1
LTS (years)	13.7	13.7	13.6	13.5	13.5	13.6
Model 5						
RSS	124.55	117.99	100.55	87.9	82.26	153.21
Improvement over Model 4	18.52%	22.31%	25.43%	29.88%	34.89%	18.01%
Seasonality (years)	1	1	1	1	1	1
Seasonality (years)	5.8	5.9	6	6	6	6
LTS (years)	13.7	13.7	13.7	13.7	13.7	13.8
Model 6						
RSS	122	116.65	99.84	87.21	81.19	151.94
Improvement over Model 5	2.08%	1.14%	0.72%	0.81%	1.32%	0.83%
Seasonality (years)	1	1	1	1	1	1
Seasonality (years)	5.5	5.8	5.9	6	6	6
Seasonality (years)	4.8	4.7	4.7	4.7	4.7	4.7
LTS (years)	13.6	13.7	13.7	13.7	13.7	13.7
Model 7						
RSS	215.39	237.51	238.92	246.12	264.55	352.1
Improvement over Model 4	-31.46%	-39.24%	-47.21%	-53.61%	-58.06%	-48.65%
Seasonality (years)	3.3	3.4	3.4	3.4	3.4	3.4
Model 8						
RSS	149.17	148.51	135.68	127.23	123.84	200.39
Improvement over Model 7	44.39%	59.93%	76.08%	93.45%	113.62%	75.71%—
Improvement over Model 5	-16.50%	-20.55%	-25.89%	-30.90%	-33.58%	-23.54%
Seasonality (years)	13.8	13.8	13.7	13.8	13.8	13.5
Seasonality (years)	5.8	5.8	5.9	6	6	3.5
Model 9						
RSS	124.55	117.99	100.56	87.91	82.26	153.21
Improvement over Model 8	19.77%	25.87%	34.93%	44.72%	50.55%	30.79%
Improvement over Model 6	-2.04%	-1.13%	-0.71%	-0.80%	-1.30%	-0.83%
Seasonality (years)	13.7	13.7	13.7	13.7	13.7	13.7
Seasonality (years)	5.8	5.9	6	6	6	6
Seasonality (years)	1	1	1	1	1	1

Table 4: **In-sample analysis. Improvement of each model over the previous ones** This Table reports, for electricity futures, the Residual Sum of Squares (*RSS*), the improvement of each model over the previous one, the seasonality, and the Long-Term Swing (LTS). The in-sample period is from 01, July, 2002 to 01, July, 2021. Each futures is indicated by the first three letters of the commodity followed by a number that indicates the order it occupies with respect to maturity.

Electricity futures			
By RSS		**By** AIC	
Model 6	616.69	Model 6	-1.15E+05
Model 5	727.72	Model 5	-1.10E+05
Model 4	887.81	Model 4	-1.03E+05
Model 3	1086.3	Model 3	-9.85E+04
Model 9	1221.9	Model 9	-9.49E+04
Model 8	1421.2	Model 8	-9.04E+04
Model 2	1516.4	Model 2	-8.86E+04
Model 7	1617.7	Model 7	-8.66E+04
Model 1	1677.1	Model 1	-8.56E+04

Table 5: **In-sample model performance ranking**. This Table reports the ranking of the models (in-sample performance) for the electricity futures. This ranking is performed by computing the mean absolute error (MAE) and the Akaike information criterion (AIC). The sample period is from 01 July, 2002 until 01 July, 2021.

Electricity	$\sum u^2$
Model 2	14.9779
Model 1	16.9664
Model 4	19.0936
Model 9	22.1958
Model 8	25.6784
Model 3	25.7090
Model 5	27.0300
Model 7	35.4735
Model 6	37.4853

Table 6: **Out-of-sample estimation ranking by cumulative error.** This Table reports the ranking of the models (out-of-sample) performance for the electricity futures. This ranking is performed by computing the forecasting errors generated by each model classified by quarters and closeness to maturity. The out-of-sample period is 2020.

Model 1	Q1	Q2	Q3	Q4	$\sum u^2$	Model 2	Q1	Q2	Q3	Q4	$\sum u^2$
EEX-2	1.2055	2.1413	0.3674	1.3225	5.0367	EEX-2	2.0545	3.0992	0.4142	0.8431	6.411
EEX-3	0.6606	0.6551	0.1898	1.0101	2.5156	EEX-3	1.0208	0.6751	0.2768	0.5825	2.5552
EEX-4	0.2953	0.5771	0.2007	0.4073	1.4804	EEX-4	0.4373	0.5767	0.2456	0.2845	1.5441
EEX-5	0.4662	1.002	0.2123	0.1181	1.7986	EEX-5	0.4779	0.8109	0.2144	0.0848	1.588
EEX-6	0.4651	1.2976	0.1799	0.7095	2.6521	EEX-6	0.3295	0.6585	0.2994	0.1577	1.4451
EEX-7	0.4536	1.3107	0.3832	1.3355	3.483	EEX-7	0.3318	0.6246	0.2256	0.2525	1.4345
$\sum u^2$	3.5463	6.9838	1.5333	4.903	**16.9664**	$\sum u^2$	4.6518	6.445	1.676	2.2051	**14.9779**
Model 3	Q1	Q2	Q3	Q4	$\sum u^2$	Model 4	Q1	Q2	Q3	Q4	$\sum u^2$
EEX-2	1.2416	3.7249	0.3788	1.4011	6.7464	EEX-2	3.8943	1.692	0.4231	0.8924	6.9018
EEX-3	0.6408	1.0618	0.1798	1.0679	2.9503	EEX-3	2.2358	0.4929	0.1947	0.4938	3.4172
EEX-4	0.235	0.3324	0.1937	0.4214	1.1825	EEX-4	0.6036	0.4493	0.3842	0.2037	1.6408
EEX-5	0.3947	0.9224	0.2241	0.1235	1.6647	EEX-5	0.2221	0.6306	0.2085	0.0601	1.1213
EEX-6	0.3827	3.3989	0.1665	0.8044	4.7525	EEX-6	0.2095	1.2193	0.5708	0.0878	2.0874
EEX-7	0.4654	6.0845	0.3213	1.5414	8.4126	EEX-7	0.3992	1.4337	1.8528	0.2394	3.9251
$\sum u^2$	3.3602	15.5249	1.4642	5.3597	**25.709**	$\sum u^2$	7.5645	5.9178	3.6341	1.9772	**19.0936**
Model 5	Q1	Q2	Q3	Q4	$\sum u^2$	Model 6	Q1	Q2	Q3	Q4	$\sum u^2$
EEX-2	3.3952	0.963	0.2581	1.4878	6.1041	EEX-2	3.9284	1.1943	0.4124	2.7776	8.3127
EEX-3	1.5521	0.2921	0.1338	2.6377	4.6157	EEX-3	2.5414	0.5269	0.2534	1.1449	4.4666
EEX-4	0.5022	0.324	0.45	3.4338	4.71	EEX-4	0.6948	0.517	0.8519	4.7894	6.8531
EEX-5	0.2681	0.398	0.2894	2.1243	3.0798	EEX-5	0.2778	0.7853	0.5408	3.4127	5.0166
EEX-6	0.4715	1.2985	0.3277	1.9527	4.0504	EEX-6	0.1756	1.8613	0.3701	2.7189	5.1259
EEX-7	0.5008	0.8847	1.2883	1.7962	4.47	EEX-7	0.2577	1.8074	1.6462	3.9991	7.7104
$\sum u^2$	6.6899	4.1603	2.7473	13.4325	**27.03**	$\sum u^2$	7.8757	6.6922	4.0748	18.8426	**37.4853**
Model 7	Q1	Q2	Q3	Q4	$\sum u^2$	Model 8	Q1	Q2	Q3	Q4	$\sum u^2$
EEX-2	1.9166	2.9067	2.1672	4.7552	11.7457	EEX-2	1.1585	1.8004	0.932	3.9299	7.8208
EEX-3	1.5141	0.6939	0.8974	1.8079	4.9133	EEX-3	0.5823	1.1154	0.8131	2.5838	5.0946
EEX-4	1.0882	0.3154	0.3026	0.6893	2.3955	EEX-4	0.1735	0.2959	0.3253	0.874	1.6687
EEX-5	1.0317	0.8005	0.2452	0.3142	2.3916	EEX-5	0.287	0.4338	0.2567	0.3011	1.2786
EEX-6	0.9453	2.9714	0.3749	0.9529	5.2445	EEX-6	0.2801	1.8526	0.3431	1.3916	3.8674
EEX-7	2.6257	4.2541	0.7216	1.1815	8.7829	EEX-7	0.6569	2.4643	0.6885	2.1386	5.9483
$\sum u^2$	9.1216	11.942	4.7089	9.701	**35.4735**	$\sum u^2$	3.1383	7.9624	3.3587	11.219	**25.6784**
Model 9	Q1	Q2	Q3	Q4	$\sum u^2$						
EEX-2	1.0262	5.3118	0.1588	1.3772	7.874						
EEX-3	0.5444	0.7975	0.3262	0.9564	2.6245						
EEX-4	0.388	0.3901	0.2395	0.8807	1.8983						
EEX-5	0.4021	0.4808	0.1749	0.2252	1.283						
EEX-6	0.4589	1.5609	0.2111	1.0958	3.3267						
EEX-7	0.5334	2.4389	0.326	1.891	5.1893						
$\sum u^2$	3.353	10.98	1.4365	6.4263	**22.1958**						

Table 7: **Out-of-sample estimation for electricity futures.** This Table reports the forecasting errors generated by each model classified by quarters and closeness to maturity. The out-of-sample period is the whole year of 2021. Each futures is indicated by the first three letters of the commodity followed by a number that indicates the order it occupies with respect to maturity.

Appendix of Figures

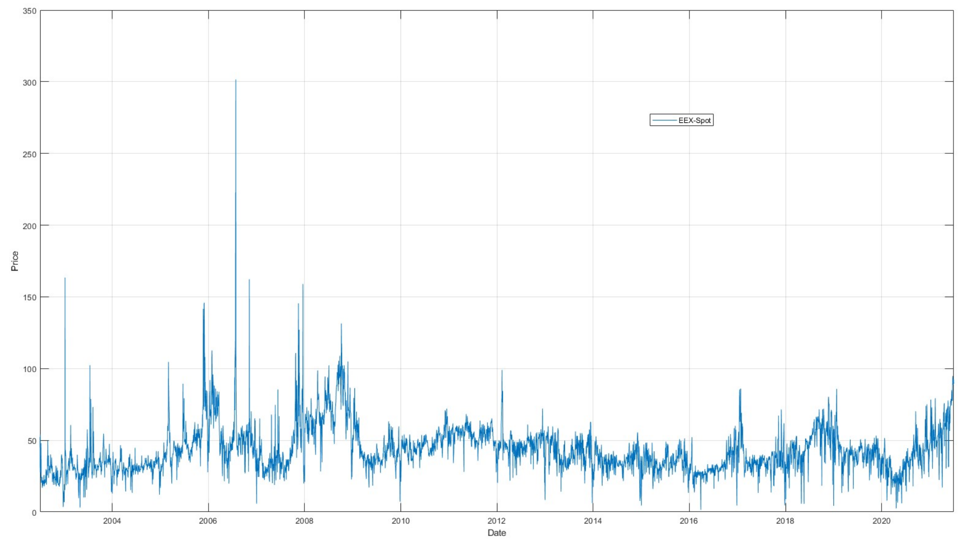

Figure 1: **Electricity spot prices from 01/07/2002 until 01/07/2021.**

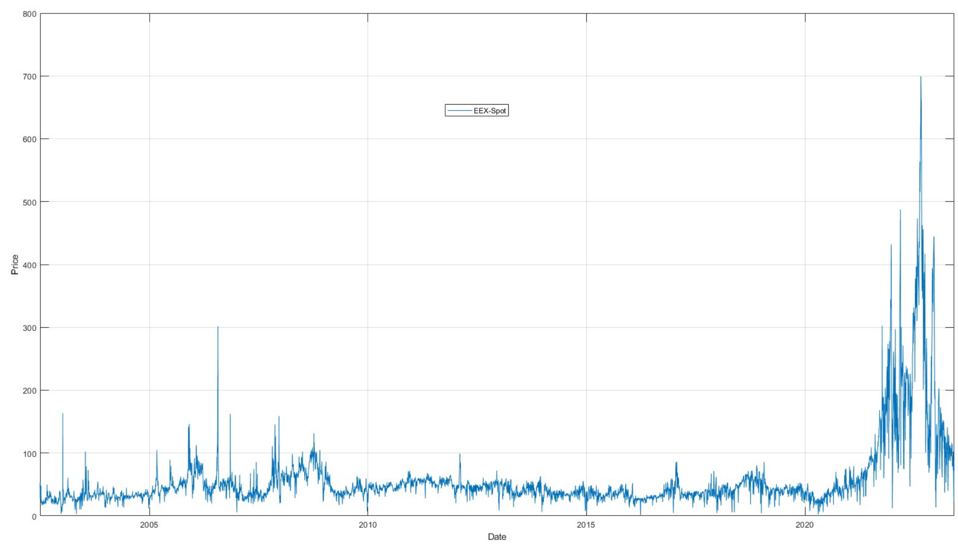

Figure 2: **Electricity spot prices from 01/07/2002 until 01/06/2023.**

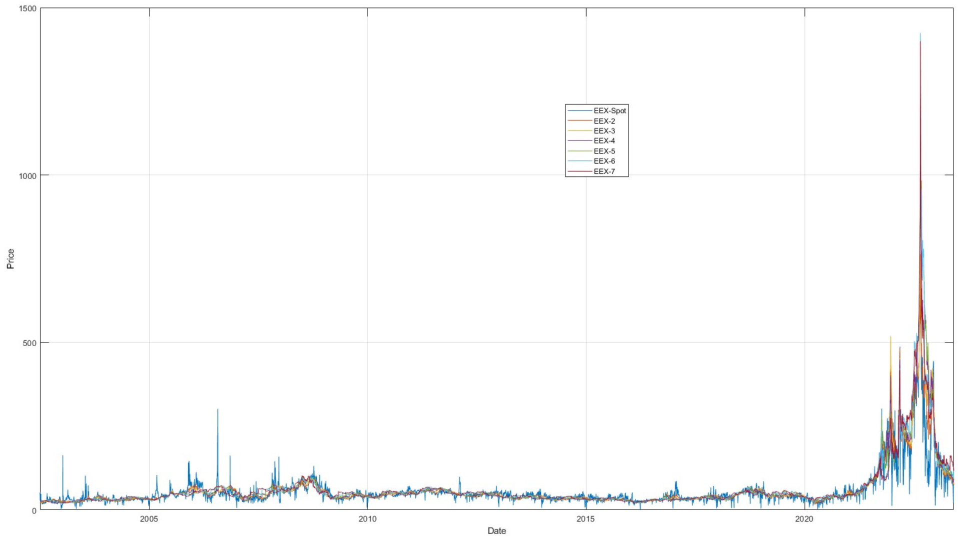

Figure 3: **Electricity spot and futures prices from 01/07/2002 until 01/06/2023.**

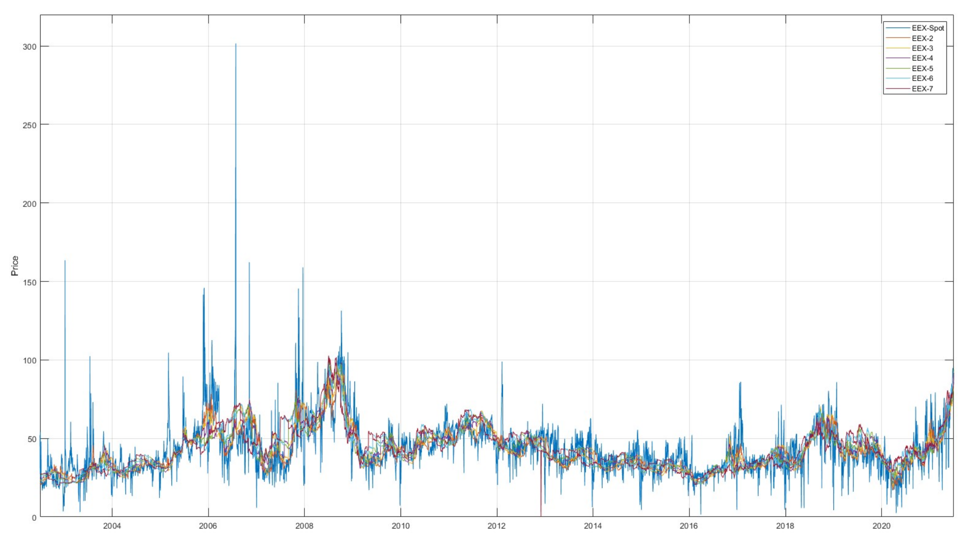

Figure 4: **Electricity spot and futures prices from 01/07/2002 until 01/07/2021.**

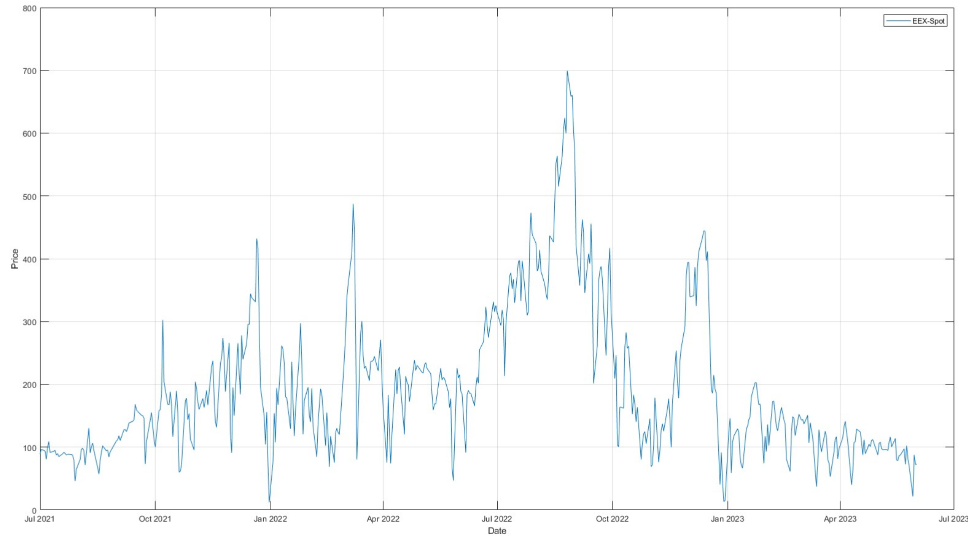

Figure 5: **Electricity spot prices from 01/07/2021 until 01/06/2023.**

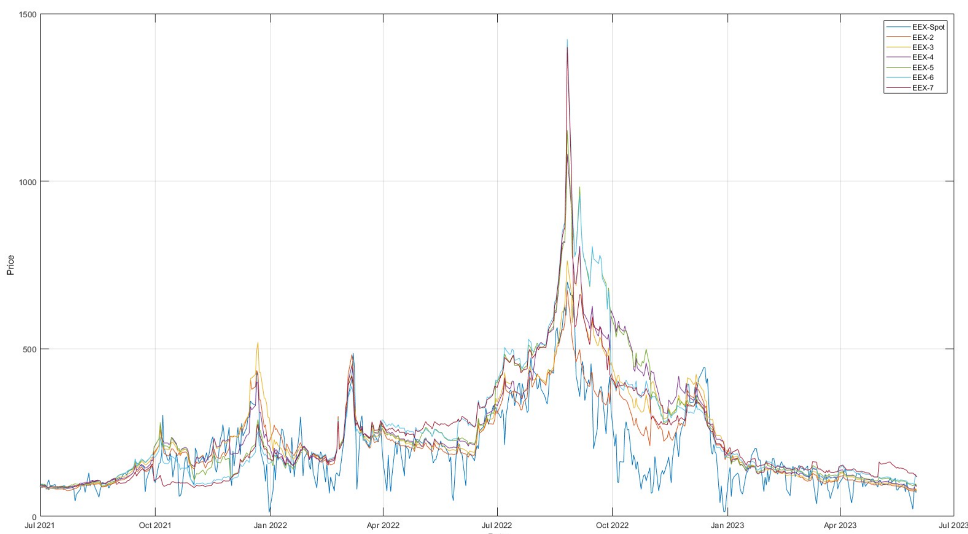

Figure 6: **Electricity spot and futures prices from 01/07/2021 until 01/06/2023.**