# Pricing of futures on $CO_2$ emission allowances: an empirical approach based on seasonal mean-reverting models

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October 12, 2023

**Abstract**

This paper analyzes the (in- and out-of-sample) performance of the one-factor model proposed in Moreno, Novales, and Platania (2019) to price futures on $CO_2$ emissions. This model assumes that the prices of futures present mean-reversion and seasonality. These features are modelled by Fourier series. We compare different specifications of this model versus the Schwartz (1997) and Lucía and Schwartz (2002) models. We use data of prices of futures on EUA (European Emission Allowances) from 2009 until 2018, traded in the Intercontinental Exchange. The main qualitative conclusions are: a) the in-sample behavior of the Moreno *et al.* (2019) model outperforms all the other one-factor models and b) there is not a clear winner forecasting model.

*Keywords:* $CO_2$ Markets, Fourier series, Seasonality, Periodic fluctuations, Long-term swing.

*The authors gratefully acknowledge comments from Michail Filippidis, Arie E. Gozluklu, Pablo Villaplana, and attendants at the 28th Forecasting Financial Markets (Rennes, France, 2023), 8th International Symposium on Environment and Energy Finance Issues (ISEFI-2022, Paris, France) and the XXIX Foro de Finanzas (Santiago de Compostela, Spain, 2022). Alexandra Balado-Alves is from Deloitte, Torre Picasso, Plaza de Pablo Ruiz Picasso 1, 28020, Madrid, Spain. E-mail: abalado@deloitte.es. Manuel Moreno is from University of Castilla-La Mancha, Department of Economic Analysis and Finance, Cobertizo San Pedro Mártir s/n, 45071 Toledo, Spain. E-mail: manuel.moreno@uclm.es. Moreno gratefully acknowledges financial support by grants PID2022-142143NB-I00 (awarded by Ministerio de Ciencia e Innovación, Spain) and SBPLY/19/180501/000172 (awarded by Junta de Comunidades de Castilla-La Mancha, co-financed by FEDER funds). The usual caveat applies.

# 1 Introduction

Carbon dioxide is a greenhouse gas that causes the increase in global temperature and that has recently reached its highest level of atmospheric concentration in the last 800,000 years leading to a large climate change. In an attempt to analyze the relevance of this asset class, this paper focuses on $CO_2$ emission allowances. In more detail, we will analyze the in- and out-of-sample empirical behavior of several continuous-time models in order to find which one fits better to the observed market price of futures on these allowances

Some authors have studied these $CO_2$ emission allowances and the prices of futures on these assets. For instance, Benz and Trück (2008), Paolella and Taschini (2008), and Seifert *et al.* (2008) performed an econometric analysis of the prices of emission allowances and study different models to determine the spot price dynamics. Chevallier (2009) analyzed the relationship between the profitability of carbon futures and changes in macroeconomic conditions. Bredin and Muckley (2011) studied how certain fundamental factors (such as economic growth, energy prices, and climatic conditions) can determine the prices of US futures. Conrad *et al.* (2011) modeled the adjustment process by which US prices respond to publications and announcements and found that the decissions of the European Commission on domestic allocation plans have a major impact on US prices.

In this paper, we focus on the one-factor model proposed in Moreno, Novales, and Platania (2019) (Moreno *et al.* (2019), from now on), which assumes that the prices of certain commodities can present mean-reversion and seasonality. The mean-reversion theory of commodity prices assumes that these prices revert to a certain long-term level: intuitively, if the current price is greater (lower) than this level, the supply of the commodity tends to increase (decrease). This model was initially proposed for energy futures and its empirical behavior for agricultural futures has been analyzed in Del Campo-Bustos and Moreno (2018).

We also consider several versions of this model in order to study the possible relevance of an additional component of seasonality and the presence of long-term oscillations. It is expected that the introduction of Fourier series in the model, which reflects seasonality and convergence to a certain long-term level, can help to estimate the prices of futures on $CO_2$ emissions.

In this paper we will perform both in- and out-of-sample empirical analysis. The in-sample behavior of the different models will be studied by applying non-linear least-squares regressions while its out-of-sample behavior will be based on the Kalman filter. All our analysis consider data from US futures (EU Allowances) from 2009 to 2018, which are traded in the Intercontinental Exchange (ICE).

The structure of this paper is as follows. Section 2 describes briefly the models that we will use in this paper. Section 3 presents the econometric methodology, section 4 describes the characteristics of the data under analysis, and section 5 provides the empirical results. Finally, section 6 summarizes the main conclusions and suggests some possible future research lines.

## 2 Models

This section introduces several one-factor models that are the starting point for the empirical analysis that will be presented subsequently.

### 2.1 Model 1 [Schwartz (1997)]

This author assumes that the spot price at time $t$ of a certain commodity, $S_t$, follows a diffusion process given by

$$dS_t = \kappa(\mu - \ln(S_t))S_t dt + \sigma dW_t$$

where $\kappa,\ \mu,\ \sigma \in \mathbb{R}$ indicate, respectively, the speed of mean-reversion, the long-term value of mean-reversion, and the diffusion coefficient of this process, and $W_t$ is a standard Wiener process. It is worth noting that this process assumes no seasonality in this price.

Let $X_t = \ln(S_t)$ be the log-spot price. Applying the Itô's Lemma, this price follows an Ornstein-Uhlenbeck stochastic process that, under the equivalent martingale measure, can be written as:

$$dX_t = \kappa(\widetilde{\alpha} - X_t)dt + \sigma d\widetilde{W_t} \tag{1}$$

where $\widetilde{\alpha} = \mu - \frac{\sigma^2}{2\kappa} - \frac{\sigma}{\kappa}\lambda$, with a constant market price of risk, $\lambda$, and $d\widetilde{W_t} = W_t + \lambda t$ is a standard Wiener process under the risk-neutral measure. Under this measure, the solution to equation (1) is given by:

$$X_s = e^{-\kappa(s-t)}X_t + (1 - e^{-\kappa(s-t)})\widetilde{\alpha} + \sigma \int_t^s e^{-\kappa(s-u)} d\widetilde{W_u}$$

Then, given the filtration $\mathscr{F}_t$, this variable follows a Gaussian distribution with conditional mean and variance:

$$\begin{aligned} \widetilde{E}[X_T|\mathscr{F}_t] &= e^{-\kappa(T-t)}\ln(S_t) + (1 - e^{-\kappa(T-t)})\widetilde{\alpha} \\ \widetilde{V}[X_T|\mathscr{F}_t] &= \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa(T-t)}) \end{aligned}$$

Therefore, applying the properties of the lognormal distribution, the forward price of the commodity at time $T$ is the expected commodity price under the martingale measure:

$$\begin{aligned} F(S_t, t, T) &= \widetilde{E}[S_T|\mathscr{F}_t] = \exp\left\{\widetilde{E}[X_T|\mathscr{F}_t] + \frac{1}{2}\widetilde{V}[X_T|\mathscr{F}_t]\right\} \\ &= \exp\left\{e^{-\kappa(T-t)}\ln(S_t) + (1 - e^{-\kappa(T-t)})\widetilde{\alpha} + \frac{\sigma^2}{4\kappa}(1 - e^{-2\kappa(T-t)})\right\} \end{aligned}$$

Taking logarithms, we get the following expression:

$$\ln(F(S_t, t, T)) = e^{-\kappa(T-t)}\ln(S_t) + (1 - e^{-\kappa(T-t)})\widetilde{\alpha} + \frac{\sigma^2}{4\kappa}(1 - e^{-2\kappa(T-t)})$$

## 2.2 Model 2 [Lucía and Schwartz (2002)]

These authors analyze the possible existence of regular patterns in the behavior of electricity prices by analyzing spot and futures prices in the Nord Pool market. This regular (seasonal) pattern in the futures curve will be modelled with a sinusoidal function. Then, the process for spot prices includes two components: a) a deterministic component that explains the regularities in the price evolution and b) a stochastic continuous-time process.[1]

In more detail, the log-spot price process can be written as:

$$\ln(S_t) = f_t + Y_t$$

where $f_t$ is a time-varying deterministic function and $Y_t$ follows a mean-reverting process, whose dynamics is given by:

$$dY_t = -\kappa Y_t dt + \sigma dW_t$$

Assuming a constant risk market price, $\lambda$, the risk-neutral version of this process is:

$$dY_t = \kappa(\alpha^* - Y_t)dt + \sigma d\widetilde{W}_t, \ \alpha^* = -\frac{\sigma\lambda}{\kappa}$$

The function $f_t$ aims to capture the behavioral components of electricity prices derived from natural regularities along time. Therefore, the seasonal behavior will be described by a process that fluctuates randomly around a constant long-term mean value. This function will include a constant plus two terms, that aim to capture, respectively, the variation in the price level distinguishing between working and non-working days and the seasonal yearly evolution of prices. In more detail, we have

$$f_t = \alpha + \beta D_t + \gamma \cos\left((t+\tau)\cdot\frac{2\pi}{365}\right)$$

where $\alpha, \ \beta, \ \gamma, \ \tau \in \mathbb{R}$ and the dummy $D_t$ equates one in week-ends or holidays and is zero otherwise. The cosine function reflects the seasonal pattern in the price evolution along the year.

Under the risk-neutral probability, the log-spot price $X_t = \ln(S_t)$ is given by:

$$X_s = f_s + e^{-\kappa(s-t)}Y_t + (1-e^{-\kappa(s-t)})\alpha^* + \sigma\int_t^s e^{-\kappa(s-u)}d\widetilde{W}_u, \ s > t$$

It is clear that the log-spot price is normally distributed and, then, the spot price follows a lognormal distribution. Finally, the price at time $t$ of a futures on this commodity that matures at $T$ is given by:

$$\begin{aligned} F(S_t,t,T) &= \widetilde{E}[S_T|\mathscr{F}_t] = \exp\left\{\widetilde{E}[X_T|\mathscr{F}_t] + \frac{1}{2}\widetilde{V}[X_T|\mathscr{F}_t]\right\} \\ &= \exp\left\{f_T + e^{-\kappa(T-t)}(\ln(S_t) - f_t) + \left(1-e^{-\kappa(T-t)}\right)\ \alpha^* + \frac{\sigma^2}{4\kappa}\left(1-e^{-2\kappa(T-t)}\right)\right\} \end{aligned}$$

[1] For simplicity, constant interest rates are considered.

### 2.3 Moreno *et al.* (2019) model

These authors present a model for commodities prices and obtain closed-form expressons for prices of different derivatives, including futures. They incorporate seasonal and cyclical fluctuations as well as long-term fluctuations in futures prices. One of their main assumptions is that futures prices converge to a long-term value that experiences periodic and smooth fluctuations during certain time periods. For cyclical and seasonal fluctuations they introduce deterministic terms in the stochastic process of the log-spot price.

In a similar way to Lucía and Schwartz (2002), these authors assume that the process for the log-spot price can be split into two components:

$$\ln(S_t) = f(t) + Y_t$$

The deterministic component $f(t)$ that represents the seasonal behavior of the commodity price is modeled with a Fourier series:

$$f(t) = \sum_{n=0}^{\infty} Re[A_n e^{inw_f t}]$$

The component $Y_t$ follows a mean-reverting process, which reverts to $z(t)$, that depends on time and captures long-term variations:

$$\begin{aligned} dY_t &= \kappa(z(t) - Y_t)dt + \sigma dW_t \\ z(t) &= \sum_{n=0}^{\infty} Re\left[B_n e^{inw_z t}\right] \end{aligned}$$

where $\kappa,\ \sigma,\ w_f,\ w_z \in \mathbb{R}^+$ and $W_t$ is a standard Wiener process. $A_n$ and $B_n$ are complex numbers: $A_n = A_{x,n} + iA_{y,n}$ and $B_n = B_{x,n} + iB_{y,n}$. If $B_n = 0$ and $A_n = 0$, this model nests that in Schwartz (1997). This model also nests the Lucía and Schwartz (2002) model if $B_n = 0$ and $f(t)$ matches the deterministic function proposed by those authors.

The futures price of a commodity is obtained using the properties of the lognormal distribution under the risk-neutral measure:

$$\begin{aligned} F(S_t, t, T) &= \tilde{E}[S_t|\mathscr{F}_t] = \exp\left\{\tilde{E}[\ln(S_t)|\mathscr{F}_t] + \frac{1}{2}\tilde{V}[\ln(S_t)|\mathscr{F}_t]\right\} \\ &= \exp\left\{f(T) + e^{-\kappa(T-t)}(\ln(S_t) - f(t)) + (1 - e^{-\kappa(T-t)})\alpha + \frac{\sigma^2}{4\kappa}(1 - e^{-2\kappa(T-t)})\right. \\ &\quad \left. + \sum_{n=1}^{\infty} Re\left[\frac{\kappa B_n}{\kappa + inw_z}(e^{inw_z T} - e^{-\kappa(T-t)+inw_z t})\right]\right\} \end{aligned}$$

that, taking logarithms, can be written as

$$\begin{aligned}\ln(F(S_t,t,T)) &= f(T)+e^{-\kappa(T-t)}(\ln(S_t)-f(t))+(1-e^{-\kappa(T-t)})\alpha+\frac{\sigma^2}{4\kappa}(1-e^{-2\kappa(T-t)}) \\ &\quad +\sum_{n=1}^{\infty} Re\left[\frac{\kappa B_n}{\kappa+inw_z}(e^{inw_zT}-e^{-\kappa(T-t)+inw_zt})\right]\end{aligned}$$

Then, the logarithm of the futures price can be split into four components:

- Spot effect, indicating a correction of the log-spot price:

$$e^{-\kappa(T-t)}\ln(S_t)$$

- Seasonal and cyclical effect, that quantifies the effect of the periodic, seasonal, and cyclical component (short- and medium-term effects):

$$f(T)-e^{-\kappa(T-t)}f(t)$$

- Volatility effect:

$$\frac{\sigma^2}{4\kappa}\left(1-e^{-2\kappa(T-t)}\right)$$

- Long-term swing effect, that is, the effect of the mean-reversion level:

$$(1-e^{-\kappa(T-t)})\alpha+\sum_{n=1}^{\infty} Re\left[\frac{\kappa B_n}{\kappa+inw_z}(e^{inw_zT}-e^{-\kappa(T-t)+inw_zt})\right]$$

Therefore, the logarithm of the futures price can be expressed as the sum of two terms:

$$\ln(F_t(S_t,t,T)) = M(S_t,t,T;\theta)+N(t,T;\theta)$$

where

$$M(S_t,t,T;\theta)=e^{-\kappa(T-t)}\ln(S_t)+(1-e^{-\kappa(T-t)})\alpha+\frac{\sigma^2}{4\kappa}(1-e^{-2\kappa(T-t)})$$

depends on the spot price and time to expiration. On the other hand, we have

$$N(t,T;\theta)=\sum_{n=0}^{\infty} Re\left[A_n(e^{inw_fT}-e^{-\kappa(T-t)+inw_ft})\right]+\sum_{n=1}^{\infty} Re\left[\frac{\kappa B_n}{\kappa+inw_z}(e^{inw_zT}-e^{-\kappa(T-t)+inw_zt})\right]$$

that represents the seasonality component, which is the novel aspect of this model with respect to previous ones. The two terms of this equation describe, respectively, the seasonal fluctuations around the long-term mean price and the time evolution of the mean-reversion level.

We will analyze seven particular cases of this model. The first four cases assume the existence of a long-term swing while the last 3 ones consider just a deterministic component that represents the seasonal behavior of the commodity price. In more detail, we have the following:

- Model 3 incorporates a single term of the Fourier series to represent the level of mean-reversion, so that the second component of the logarithm of the futures price is given by:

$$N(t,T;\theta) = Re\left[\frac{\kappa B}{\kappa + iw_z}(e^{iw_z T} - e^{-\kappa(T-t)+iw_z t})\right]$$

- Model 4 adds a Fourier series representation with a single frequency for $f(t)$, with the goal of capturing the cyclical and seasonal component, so the second component will be:

$$N(t,T;\theta) = Re[A(e^{iw_{f,1}T} - e^{-\kappa(T-t)+iw_{f,1}t})] + Re\left[\frac{\kappa B}{\kappa + iw_z}(e^{iw_z T} - e^{-\kappa(T-t)+iw_z t})\right]$$

- Model 5 considers two different frequencies in the series $f(t)$, aiming to capture fluctuations in the business cycle and short-term seasonality. So, we have:

$$N(t,T;\theta) = \sum_{l=1,2} Re[A_l(e^{iw_{f,l}T} - e^{-\kappa(T-t)+iw_{f,l}t})] + Re\left[\frac{\kappa B}{\kappa + iw_z}(e^{iw_z T} - e^{-\kappa(T-t)+iw_z t})\right]$$

- Model 6 considers three frequencies in $f(t)$ trying to improve the fit to observed data. In this case, we have:

$$N(t,T;\theta) = \sum_{l=1,2,3} Re[A_l(e^{iw_{f,l}T} - e^{-\kappa(T-t)+iw_{f,l}t})] + Re\left[\frac{\kappa B}{\kappa + iw_z}(e^{iw_z T} - e^{-\kappa(T-t)+iw_z t})\right]$$

- Model 7 considers a single frequency for $f(t)$ and does not consider the representation of the level of mean-reversion. Therefore, we have:

$$N(t,T;\theta) = Re[A(e^{iw_{f,1}T} - e^{-\kappa(T-t)+iw_{f,1}t})]$$

- Model 8 incorporates two frequencies in the $f(t)$ series that capture the fluctuations of the business cycle and short-term seasonality:

$$N(t,T;\theta) = \sum_{l=1,2} Re[A_l(e^{iw_{f,l}T} - e^{-\kappa(T-t)+iw_{f,l}t})]$$

- Model 9 adds a third frequency for $f(t)$ trying to capture better the seasonality, so we have:

$$N(t,T;\theta) = \sum_{l=1,2,3} Re[A_l(e^{iw_{f,l}T} - e^{-\kappa(T-t)+iw_{f,l}t})]$$

Moreno *et al.* (2019) estimated empirically models 1 to 5 using prices of three energy futures and found that their models (3 to 5) outperformed models 1 and 2.

# 3 Econometric methodology

We describe now the optimization problem and the econometric methodology that will be implemented to estimate the parameters of the different models that were presented previously.

## 3.1 Optimization problem and in-sample estimation

The optimization problem to be solved is given by

$$\min(SR(\tilde{\theta};\tilde{\theta}_m)) = \sum_{t=1}^{T}\sum_{j=1}^{\kappa}(P_{jt} - \beta^{'}\eta_{jt})^{'}W(P_{jt} - \beta^{'}\eta_{jt})$$

where $k$ indicates the number of parameters to be estimated, $W = I_k$ and

$$\beta^{'}\eta_{j,t} = \sum_{i=1}^{8}\beta_i\eta_{ij,t}$$

We consider the variable $P_t = \ln(F(S_t, t, T)) - e^{\kappa(T-t)}\ln(S_t)$ that is stationary as it tends to 0 when $t \to T$. We have:

$$P_t = \sum_{i=1}^{10}\beta_i\eta_{it} + u_t$$

where $u_t$ denotes the error term. Then, $P_t$ and $\eta_{it},\ i = 1, \cdots, 10$ are non-linear functions of the structural parameters.

The price of each commodity presents a common structure for each futures but, in addition, there may be specific seasonal and cyclical components for each maturity of the corresponding futures. Then, for all the models, we have

$$\eta_{1t} = 1 - e^{-\kappa(T-t)}, \quad \eta_{2t} = \frac{1}{4\kappa}(1 - e^{-2\kappa(T-t)})$$

while $\eta_{it},\ i = 3, \cdots, 10$, present a different specification for each model.

We want to separate the long-term common component and the specific components of the different maturities. Then, we need to impose a common level of mean-reversion for all the maturities but allowing seasonal and cyclical components to differ across maturities.

For each model, we have the following parameters and Fourier components:

- Model 1: Schwartz (1997):

$$\beta_1 = \tilde{\alpha} = \mu - \frac{\sigma^2}{2\kappa} - \frac{\sigma\lambda}{\kappa},\ \beta_2 = \sigma^2,\ \beta_i = 0,\ i = 3, \cdots, 8$$

- Model 2: Lucía and Schwartz (2002). This model incorporates a seasonality component:

$$\begin{aligned}\beta_1 &= \tilde{\alpha} = \mu - \frac{\sigma\lambda}{\kappa},\ \beta_2 = \sigma^2, \quad \beta_3 = \gamma,\ \beta_i = 0,\ i = 4, \cdots, 8 \\ \eta_{3t} &= \cos\left((T+\varphi)\frac{2\pi}{260}\right) - e^{\kappa(T-t)}\cos\left((t+\varphi)\frac{2\pi}{260}\right)\end{aligned}$$

We now study the remaining models in which we have always $\beta_1 = \alpha,\ \beta_2 = \sigma^2$. Moreover, we have the following:

- Model 3 incorporates long-term oscillations, where

$$\begin{aligned}\beta_3\eta_{3t} + \beta_4\eta_{4t} &= Re\left[(B_x + iB_y)\frac{\kappa}{\kappa + iw_z}(e^{iw_zT} - e^{-\kappa(T-t)+iw_z\kappa t})\right] \\ \beta_i &= 0,\ i = 5, \cdots, 10\end{aligned}$$

- Model 4 includes long-term oscillations and a seasonality component. So, we have:

$$\begin{aligned}\beta_3\eta_{3t} + \beta_4\eta_{4t} &= Re\left[(B_x + iB_y)\frac{\kappa}{\kappa + iw_z}(e^{iw_zT} - e^{-\kappa(T-t)+iw_z\kappa t})\right] \\ \beta_5\eta_{5t} + \beta_6\eta_{6t} &= Re\left[(A_{x,1} + iB_{y,1})e^{iw_{f,1}t}\right] \\ \beta_i &= 0,\ i = 7, 8, 9, 10\end{aligned}$$

- Model 5 incorporates two terms in the Fourier series, related to the cyclical component, and a term in the long-term mean. In this case, we have

$$\begin{aligned}\beta_3\eta_{3t} + \beta_4\eta_{4t} &= Re\left[(B_x + iB_y)\frac{\kappa}{\kappa + iw_z}(e^{iw_zT} - e^{-\kappa(T-t)+iw_z\kappa t})\right] \\ \sum_{i=5}^{8}\beta_i\eta_{it} &= \sum_{l=1,2} Re\left[(A_{x,l} + iA_{y,l})e^{iw_{f,l}t}\right] \\ \beta_9 &= \beta_{10} = 0\end{aligned}$$

Models 4 and 5 incorporate more than one Fourier series. The lowest frequency is related to long-term oscillations in the level of mean-reversion and the remaining frequencies define the cyclical or seasonal components of futures prices.

We present now models 6 to 9, which extend models 3 to 5:

- Model 6 adds an additional seasonal component to model 5. The Fourier components are now given by

$$\begin{aligned}\beta_3\eta_{3t} + \beta_4\eta_{4t} &= Re\left[(B_x + iB_y)\frac{\kappa}{\kappa + iw_z}(e^{iw_zT} - e^{-\kappa(T-t)+iw_z\kappa t})\right] \\ \sum_{i=5}^{10}\beta_i\eta_{it} &= \sum_{l=1,2,3} Re\left[(A_{x,l} + iA_{y,l})e^{iw_{f,l}t}\right]\end{aligned}$$

- Model 7 is equal to model 4 eliminating the long-term oscillations. Then, it includes just one seasonality component. So, we have:

$$\begin{aligned} \beta_5\eta_{5t} + \beta_6\eta_{6t} &= Re\left[(A_{x,1} + iB_{y,1})e^{iw_{f,1}t}\right] \\ \beta_i &= 0,\ i = 7, 8, 9, 10 \end{aligned}$$

- Model 8 incorporates two terms in the Fourier series, related to the cyclical component. In this case,

$$\begin{aligned} \sum_{i=5}^{8} \beta_i\eta_{it} &= \sum_{l=1,2} Re\left[(A_{x,l} + iA_{y,l})e^{iw_{f,l}t}\right] \\ \beta_3 &= \beta_4 = \beta_9 = \beta_{10} = 0 \end{aligned}$$

- Model 9 adds one more seasonal component to model 8, so the Fourier components are now

$$\sum_{i=5}^{10} \beta_i\eta_{it} = \sum_{l=1,2,3} Re\left[(A_{x,l} + iA_{y,l})e^{iw_{f,l}t}\right]$$

### 3.2 Out-of-sample estimation

We will apply the Kalman filter to study the forecasting performance of all the models. Following Harvey (1989), this filter is a recursive algorithm, which allows to evaluate numerically a likelihood function to estimate certain parameters, generating estimates of the unobservable variables included in the model. A limitation of this technique is that it assumes that no observable variable can affect unobservable states or variables.

To apply this algorithm we need to write the model in the form of state space, such that:

$$\begin{aligned} \xi_{t+1} &= F\xi_t + \upsilon_{t+1} \\ y_t &= A'x_t + H'\xi_t + \omega_t \end{aligned}$$

These equations are named state and observation equations, respectively. The vector $\xi_t$ includes the unobservable variable while $y_t$ and $x_t$ include the observable variables. In more detail, $x_t$ contains exogenous variables.

Our goal is to predict $\xi_{t+1}$ e $y_{t+1}$ with the set of information available at time $t$, $Y_t$, the prediction being $\hat{\xi}_{t+1|t} = E(\xi_{t+1}|Y_t)$.

All the previous models assume that there is a relationship between spot and futures prices, although the spot market tends to present more abrupt changes than the futures market. This is because, in the spot market, the closing prices are liquidation prices while, in the futures market, average prices along the last minutes of the session are considered. For this reason, we assume that spot prices can incorporate a noise component.

To perform the forecasting analysis of the one-factor models, the Kalman filter technique will be used, where the representation in the state space is as follows:

$$\begin{aligned} \ln(S_{t+1}) &= G + F\ln(S_t) + \upsilon_{t+1} \\ \ln(F(S_t,t,T)) &= A' x_t + H' \ln(S_t) + \omega_t \end{aligned}$$

where $E(\upsilon_t \upsilon_t') = Q$ and $E(\omega_t \omega_t') = R$, being $Q$ and $R$ constants.

We take the log-spot price at time $t+1$ as an unobservable variable in the state space representation and we proceed as follows: at time $t$, the filtered spot price, $S_t$, and the estimates of $\hat{F}$ and $\hat{G}$ are used to predict $S_{t+1}$, which will be used to estimate the observation equation one period later, $\hat{F}_{t+1}$.

The observation equations, specific for each model, are the following:

- **Model 2** : Lucía and Schwartz (2002):

$$\ln(F(S_t,t,T)) = \begin{pmatrix} \alpha & \frac{\sigma^2}{4\kappa} & \gamma & \gamma \end{pmatrix} \begin{pmatrix} 1 - e^{-\kappa(T-t)} \\ 1 - e^{-2\kappa(T-t)} \\ \cos\left((T+\varphi)\frac{2\pi}{260}\right) \\ -e^{-\kappa(T-t)}\cos\left((t+\varphi)\frac{2\pi}{260}\right) \end{pmatrix} + e^{-\kappa(T-t)}\ln(S_t) + \omega_t$$

- The remaining models can be summarized in a single matrix since, for each model, we get a particular case of a more general matrix:

$$\ln(F(S_t,t,T)) = A\, x_t' + e^{-\kappa(T-t)}\ln(S_t) + \omega_t$$

  where

$$A = \begin{pmatrix} \alpha \ \frac{\sigma^2}{4\kappa} \ B_x \ B_y \ A_{x,1} \ A_{y,1} \ A_{x,2} \ A_{y,2} \ A_{x,3} \ A_{y,3} \end{pmatrix}$$

  and

$$x_t' = \begin{pmatrix} 1 - e^{-\kappa(T_i-t)} \\ 1 - e^{-2\kappa(T_i-t)} \\ \kappa\cos(\omega_z T_i) + \omega_z\sin(\omega_z T_i) - e^{-\kappa(T_i-t)}[\kappa\cos(\omega_z t) + \omega_z\sin(\omega_z t)] \\ -\kappa\sin(\omega_z T_i) + \omega_z\cos(\omega_z T_i) - e^{-\kappa(T_i-t)}[\omega_z\cos(\omega_z t) - \kappa\sin(\omega_z t)] \\ \cos(\omega_f T_i) - e^{-\kappa(T_i-t)}\cos(\omega_f t) \\ e^{-\kappa(T_i-t)}\sin(\omega_f t) - \sin(\omega_f T_i) \\ \cos(\omega_{f2} T_i) - e^{-\kappa(T_i-t)}\cos(\omega_{f2} t) \\ e^{-\kappa(T_i-t)}\sin(\omega_{f2} t) - \sin(\omega_{f2} T_i) \\ \cos(\omega_{f3} T_i) - e^{-\kappa(T_i-t)}\cos(\omega_{f3} t) \\ e^{-\kappa(T_i-t)}\sin(\omega_{f3} t) - \sin(\omega_{f3} T_i) \end{pmatrix}$$

Thus, each model is based on the following columns of the matrices $A$ and $x_t$:

- Model 1: Columns 1 and 2.
- Model 3: Columns 1 to 4.
- Model 4: Columns 1 to 6.
- Model 5: Columns 1 to 8.
- Model 6: Columns 1 to 10.
- Model 7: Columns 1, 2, 5, and 6.
- Model 8: Columns 1, 2, and 5 to 8.
- Model 9: Columns 1, 2, and 5 to 10.

We will perform the forecasting analysis for two different years, 2016 and 2017. We start predicting the first quarter of 2017. For this, we use daily data from 2016 to calibrate each model, except the frequency parameters for which we use the complete sample until the end of 2016. Once the model is calibrated, we obtain the one-day ahead forecasts for futures prices during the first quarter of 2017. Later, we recalibrate the parameters of each model using data from April, 2016 to March, 2017. For the frequency parameters we use the entire sample until March, 2017. And we forecast the second quarter of 2017. We continue this process until we get one-day ahead predictions for futures prices along the four quarters of 2017.

We perform the same procedure but using 2015 and 2016 data instead of just 2016. Thus, we forecast the first quarter of 2017 by using data from both years to calibrate the model and data from the whole sample until the end of 2016 to estimate the frequency parameters. The second quarter of 2017 is forecasted by using data from April, 2015 to March, 2017. And so on until we have predictions for the four quarters of 2017.

We apply the same method to forecast the year 2016, using data from either 2015 or 2014 and 2015. We will compare our forecasted values versus observed data and we obtain the forecasting errors, that will be discussed accordingly.

# 4 Data

As mentioned above, we will use data for futures on $CO_2$ EUA (European Emission Allowances). These futures have monthly maturities and, in each contract, 1,000 permits of $CO_2$ emissions are traded. Each permit allows one ton of carbon dioxide to be emitted and it is traded in euros and cents per metric ton. The minimum tick is 0.01€ per ton or, equivalently, 10€ for each futures.

The futures expires at the last Monday of the corresponding month.[2] The closing price is computed as the average of the quoted prices during the daily closing period (16:50:00 - 16:59:59 hours, UK local time), or with quoted liquidation prices if the liquidity is low. Contracts are physically settled and delivered by transferring the EU allowances from the seller account to the buyer account in the Union Registry. All the transfers go through the account of the clearing member and ICE Clear Europe. The delivery is done three days after the last trading day.

Data series have been obtained from Datastream and closest-to-maturity futures will be used. For all the maturities, the sample period ends in March 9, 2018. The beginning of the sample period depends on the considered maturity. Specifically, we have the following starting days:

- Spot price and first and second closest-to-maturity futures: October 27, 2009.
- Third and fourth closest-to-maturity futures: July 30, 2010.
- Fifth closest-to-maturity futures: December 29, 2010.
- Sixth and seventh closest-to-maturity futures: April 3, 2012.

Table 1 provides several descriptive statistics for the prices of the closest-to-maturity futures and the spot price. We see that the mean futures price ranges between 6.91 and 8.55, an interval that includes the mean spot price. We also see that the volatility increases in the five closest maturities and is smaller in the sixth and seventh closest-to-maturity futures. This fact can be explained by the seasonality, that implies that the periods with higher amount of information relative to the level of production also present the highest volatility in all the maturities. Then, in the futures that mature after these periods, we can observe intervals of volatility increasing with maturity.

[INSERT TABLE 1 AROUND HERE]

In all the cases, we see a positive skewness, indicating that extreme high prices have higher probability than low ones. This can be also checked in the extreme values as the maximum values are more extreme than minimum ones. The kurtosis ranges between 2.27 and 5.80 and increases with maturity except in maturitiess 6 and 7.

Figure 1 includes the time series for the spot and futures prices and shows a similar evolution in all the maturities. From mid 2015 on, the time evolution in all the series is so similar that they are almost indistinghuisible. These futures reach maximum values around, respectively, May, 2011 and April, 2013. In addition, the market seems to be in contango in most of the sample period.

[2] If the last Monday is a non-working day or there is a non-working day in the 4 days after the last Monday, the last trading day will be the penultimate Monday of the delivery month.

[INSERT FIGURE 1 AROUND HERE]

Finally, we have studied the stationarity of these prices (in levels and in first differences) by computing the augmented Dickey-Fuller statistic. Table 2 presents the results and shows that the spot and futures prices include a unit root while all the first differences of these series are stationary.

[INSERT TABLE 2 AROUND HERE]

# 5 Empirical analysis

This section includes the empirical analysis and the results obtained when estimating the parameters of the models described above. We start with the in-sample analysis of the nine one-factor models.

## 5.1 In sample analysis

Table 3 includes the in-sample estimates of the parameters for each model and three measures of goodness of fit: the minimum of the sum of squared errors, $\sum_t \min SCR(\hat{\theta}_t) = \sum_t \hat{u}_t$, its standard deviation, $\frac{1}{n}\sum_{t=1}^{n} \hat{u}_t^{1/2}$, and the absolute mean error, $\frac{1}{n}\sum_{t=1}^{n} |\hat{u}_t|$. These estimates have been obtained using the entire sample available for each futures. It can be seen that the model 9 provides the smallest sum of squared errors, although the most notable improvement occurs when moving from model 2 to model 3. This suggests that, to explain futures prices, it is necessary to introduce a cyclical component, either long- or short-term. It should also be noted that the models that exhibit long-term oscillations have $SCR$ very similar to the models that only incorporate seasonality.

[INSERT TABLE 3 AROUND HERE]

Table 4 provides the sums of squared errors for each maturity and its comparison for each model. We can conclude that including the Fourier terms in the process that follows the log-futures price improves the adjustment to market observed data, since the sum of squared errors decreases. The highest improvement appears in models 3 and 7, which incorporate long-term oscillations and a cyclical component, respectively. That is, the inclusion of the additional cyclical components of models 4, 5, 8, and 9 improves the adjustment but this improvement is not very high.

[INSERT TABLE 4 AROUND HERE]

We now discuss the main results obtained with each model:

- Model 2 includes a seasonal component but the improvement with respect to model 1 is marginal (0.83%). This fact does not indicate that futures prices do not present seasonality but that this is already incorporated in the price, since it is present in spot prices.

- Model 3 incorporates oscillations in the level of mean-reversion through a Fourier term and this incorporation leads to an improvement of almost 44% and 45% with respect to, respectively, models 2 and 1. According to this model, the estimated frequency of long-term fluctuations is approximately 20.5 years.

- Model 4 includes a seasonal component and a Fourier term with a frequency for the seasonal and cyclical components. The improvements produced by this model are, respectively, 51.11%, 50.71%, and 11.07% with respect to models 1, 2, and 3. In this model, the long-term oscillations in the level of mean-reversion are around 13 years, significantly lower than the frequency obtained for the previous model.

- Model 5 incorporates a second frequency in the Fourier series and produces an improvement of 52.21%, 51.81%, 13.06% and 2.25% with respect to the first four models. In this case, the estimated frequency for long-term oscillations is 14 years. The periodic component suggests a period of between 3.5 and 4.5 years, except for the second term that has a period of 8.5 years. Finally, the second frequency ranges between 4 and 10 years.

- The improvement in model 6 with respect to model 5 is not very relevant, which indicates that the effect of the last seasonal component is marginal when it comes to explain futures prices. The estimated frequency for long-term oscillations is 13.5 years and the seasonal frequencies range between 1.5 and 7 years.

- Model 7 is similar to the model 4, with the difference that in this model long-term oscillations are not incorporated in the level of mean-reversion. This model produces improvements of 47.48%, 47.04% and 4.45% with respect to models 1, 2, and 3, respectively. With respect to models 4 and 5, there is a worsening of 7.45% and 9.90%, respectively.

- Model 8 is similar to model 5 but it does not incorporate the long-term swing. The improvements that are produced are 52.29%, 51.89%, 13.21%, 2.40%, 0.17%, and 9.17% with respect to, respectively, models 1 to 6.

- Finally, the model 9 presents the smallest sum of squared errors although the improvement with respect to the model 8 is not very high.

Therefore, we can conclude that the ranking of models (from least to most suitable) is: 1 - 2 - 3 - 7 - 4 - 5 - 8 - 6 - 9. Although the highest relative improvement between models occurs in models 3 and 7, when including long-term oscillations, the model 9 presents the lowest SCR, so including seasonality also helps greatly to estimate the price of futures on $CO_2$ emissions.

Figure 2 shows the fitted futures prices for the nine estimated models and the observed futures prices. For the fifth maturity, all the models provide a good adjustment, being better in models 7, 4, and 6, and worse in models 1 and 2. Although we must emphasize that the model with the best fit is not always the same but depends on the period under analysis. For this reason, several periods will be studied later. Figure 3 shows that the spot effect does not exactly reproduce the level of the futures price but they present a very similar behavior.

[INSERT FIGURES 2 and 3 AROUND HERE]

Figure 4 shows the decomposition of the four effects of the model 5 for the second nearest maturity. These effects are: spot effect, volatility effect, effect of periodic (seasonal and cyclical) component, and effect of long-term oscillations. In figure 5, the level of mean-reversion (long-term swing) is observed in the model 5 for the five maturities of the chosen futures. We can see a certain synchronization among the levels obtained for the different maturities.

[INSERT FIGURES 4 and 5 AROUND HERE]

We analyze now these models along two subperiods, since the Kyoto Protocol expired at the end of 2012 and its continuation was signed at the 2012 Doha summit. Then, we consider a first subperiod sicne the first available data for each futures until the end of 2012 and a second subperiod that starts in 2013 and ends in March, 2018.

Table 5 includes the estimates and the measures of goodness of fit for the first subperiod. Models 5 and 9 improve the results from the other models, being the model 9 the one with the highest gain, 31.90%, with respect to the model 2.

[INSERT TABLE 5 AROUND HERE]

Table 6 shows, for each maturity, the SCR, the seasonality period, and the improvement of each model. It can be seen that the improvements with respect to models 1 and 2 are not so high as when we consider the entire sample. Even so, certain models (such as 4 or 7) show notable improvements. The frequency of long-term oscillations is estimated around 10 years for models 3, 4, and 5, and around 17 years for the model 6.

[INSERT TABLE 6 AROUND HERE]

Figure 6 shows the fitted prices provided by the different models for the second maturity (the one with more available data) and the observed futures prices. In this case, models 5 and 9 are those that seem to produce a better fit.

[INSERT FIGURE 6 AROUND HERE]

For the second subperiod, Table 7 shows the estimated parameters and the measures of goodness of fit. The model 9 presents the lowest SCR, with an improvement of 41.07% over the model 1. Models 4, 5, 6, and 8 have a very similar SCR, so all these models could be considered reasonable when estimating these futures prices. In this subperiod, the highest improvement, 39.69%, occurs from model 2 to model 3.

[INSERT TABLE 7 AROUND HERE]

Table 8 shows the improvements and seasonality periods of each model for each maturity. We see that the frequency of long-term oscillations fluctuates between 10 and 13.5 years depending on the model. The highest improvements with respect to model 2 occur in models 3 and 7.

[INSERT TABLE 8 AROUND HERE]

Figure 7 illustrates that the model 6 provides the worst fit at the beginning of the sample but does not provide the highest sum of squared errors. This is because, although it fits poorly at the beginning, it adjusts quite well later, so that other models with an average adjustment along the sample end up having a higher SCR.

[INSERT FIGURE 7 AROUND HERE]

Table 9 summarizes the ranking of models (from lowest to highest SCR) along with the ranking of the estimated models on a yearly basis. We can see that the model 9 has the lowest SCR although the highest improvement occurs in other models, such as 4, 8, or 3.

[INSERT TABLE 9 AROUND HERE]

Then, we can say that the model 9 presents the lowest SCR for all the periods under analysis. However, in many cases, this SCR is similar to that in other models. Therefore, given the cost of adding more frequencies, one might think that one of the other models would also be appropriate. It is necessary to highlight that, in all the cases, models 1 and 2 do present a sum of squared errors that is significantly higher than in the remaining models, so including the seasonal components does significantly improve the estimates.

## 5.2 Forecasting analysis

This subsection studies whether the aforementioned models help to anticipate changes in the market prices of futures on $CO_2$ emission allowances. Then, we compare now all the models, focusing on their sum of squared forecasting errors.

Tables 10 to 13 summarize the main results. Table 10 shows the forecasting results for the year 2016 using data from 2015. The model 7 provides the lowest forecasting error but it is not the model with the best in-sample fit (model 9). Analyzing the different terms, the model 7 also presents the smallest error for the first term, the model 6 is the best one for the second term and the model 9 is the best one for the last two terms. The model 3 has the highest total error.

[INSERT TABLE 10 AROUND HERE]

If we use the information of 2014 and 2015, Table 11 shows that the model 3 presents the smallest error. It can be seen that, using the information of both years, the errors increase considerably. On a three months basis, models 5, 3, 4, and 9 are, respectively, the best ones for the different terms of 2016. Finally, the model 8 shows the highest overall error.

[INSERT TABLE 11 AROUND HERE]

Table 12 shows the results for 2017. Clearly, the model 4 is the best one as it presents the lowest forecasting errors. Once again, the model with the best in-sample behavior (model 9) does not show the best out-of-sample performance. The highest forecasting errors are obtained in the first and fourth terms. The model 3 provides the smallest errors in the first two terms. Models 1 and 4 obtain the best fit in the third term and the model 4 is the best one along the last term. The model 8 provides the worst overall fit.

[INSERT TABLE 12 AROUND HERE]

Using the information of 2015 and 2016, Table 13 shows that the model 5 provides the smallest error but the model 3 provides a similar result. The best performance for all the models happens in the last term. Looking at the different terms, models 5, 6, 3, and (again) 5 are the best ones for each term. As previously, the model 8 presents the worst global performance.

[INSERT TABLE 13 AROUND HERE]

Therefore, we have illustrated that the model with the best in-sample behavior is not necessarily the best forecasting model. Moreover, we can conclude that there is not a clear winner model from an out-of-sample point of view.

# 6 Conclusions

$CO_2$ emissions are currently a very important issue with relevant implications in the global context due to the pernicious effects of these emissions on the atmosphere and global warming. This paper focuses on the analysis of futures on $CO_2$ emission allowances and tries to find the most adequate model in order to make both in- and out-of-sample estimations. To this aim, we have compared different specifications of the Moreno *et al.* (2019) model versus the models proposed in Schwartz (1997) and Lucía and Schwartz (2002). In addition, we have also introduced a two-factor model.

Moreno *et al.* (2019) proposed initially their model for prices of energy futures, which present long-term flctuations in the level of mean-reversion and seasonality, using Fourier series to represent these seasonal and cyclical components. The possible advantages of this model come from its flexibility to allow long-term oscillations in the level of mean-reversion withouth imposing any constraints on the frequencies to be estimated in the Fourier series. This model also provides closed-form expressions for the prices of different derivatives and, specifically, for futures prices.

We have used EUA data from 2009 until 2018 and we have estimated the models proposed in Schwartz (1997), Lucía and Schwartz (2002), and seven versions of the model in Moreno *et al.* (2019). These one-factor models have been estimated *in-sample* by applying non-linear least-squares regressions.

We have performed the estimations by considering different alternatives: a) for the whole sample, b) from the first available data until the end of 2012 (final moment of the Kyoto protocol), c) from 2013 until March, 2018, period in which the protocol is prolonged, thanks to the 2012 Doha summit and d) yearly estimates.

The in-sample empirical analysis shows that the model 9 (including three seasonal components and no frequency in the long-term oscillations in the level of mean-reversion) is the most adequate model in all the periods under study. However, in several cases, the improvement with respect to models 3 to 8 is not very noticeable. We can highlight that all the versions of the Moreno *el al.* (2019) model outperform the models in Schwartz (1997) and Lucía and Schwartz (2002). Then, we conclude that the introduction of seasonality with the Fourier series improves the in-sample estimation.

Out-of-sample estimates are obtained for the period 2016-2017 by applying the Kalman filter. The different versions of the Moreno *et al.* (2019) model do not seem so adequate as in the in-sample analysis since the sum of squared errors do not differ significantly from those obtained for the models in Schwartz (1997) and Lucía and Schwartz (2002). Models 7 and 4 provide the best forecasting behavior for respectively, 2016 and 2017. Hence, we conclude that the different cases embedded in Moreno *et al.* (2019) are adequate to perform the in-sample estimations but they are not so good for prediction purposes.

As future research lines, we can analyze the empirical behavior of the one-factor models analyzed in this paper to explain prices in other commodity markets as, for instance, electricity. A final avenue for further research can be to consider two-factor models that include the different versions of the Moreno *et al.* (2019) model.

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# Appendix of Tables

Descriptive statistics of the futures on $CO_2$ emissions.

| | Beginning of the sample | Mean | Standard deviation | Coeff. of variation | Skewness | Kurtosis | Maximum | Minimum |
|---|---|---|---|---|---|---|---|---|
| Spot | 27, October, 2009 | 8.13 | 3.64 | 0.45 | 0.95 | 2.58 | 16.8 | 2.7 |
| c1 | 27, October, 2009 | 8.29 | 3.84 | 0.46 | 0.97 | 2.60 | 17.4 | 2.79 |
| c2 | 27, October, 2009 | 8.55 | 4.04 | 0.47 | 0.98 | 2.58 | 18.25 | 2.89 |
| c3 | 30, July, 2010 | 8.21 | 3.96 | 0.48 | 1.44 | 4.04 | 19.67 | 3.01 |
| c4 | 30, July, 2010 | 8.52 | 4.24 | 0.50 | 1.45 | 4.03 | 20.9 | 3.16 |
| c5 | 29, December, 2010 | 8.28 | 4.00 | 0.48 | 1.81 | 5.80 | 22.01 | 3.28 |
| c6 | 3, April, 2012 | 6.91 | 1.67 | 0.24 | 0.34 | 2.27 | 11.39 | 3.48 |
| c7 | 3, April, 2012 | 7.05 | 1.82 | 0.26 | 0.53 | 2.49 | 11.93 | 3.71 |

**Table 1**: Descriptive analysis of spot prices and futures prices on emissions of $CO_2$. We show mean, standard deviation, coefficient of variation, skewness, kurtosis, maximum, and minimum. For all the prices, the sample ends at 9, March, 2018.

Dickey-Fuller statistics.

| | ADF (levels) | | ADF (differences) | |
|---|---|---|---|---|
| | Value | Statistic | Value | Statistic |
| Spot | -0.83 | -3.99 | -45.61 | -3.99 |
| c1 | -2.53 | -3.99 | -49.42 | -3.99 |
| c2 | -2.54 | -3.99 | -49.31 | -3.99 |
| c3 | -2.29 | -3.99 | -48.41 | -3.99 |
| c4 | -2.14 | -3.99 | -46.49 | -3.99 |
| c5 | -2.03 | -3.99 | -45.36 | -3.99 |
| c6 | -3.21 | -3.99 | -41.12 | -3.99 |
| c7 | -3.19 | -3.99 | -40.72 | -3.99 |

**Table 2**: Augmented Dickey-Fuller statistic (for levels and first differences) for the spot price and futures c1 to c7.

Parametric in-sample estimations, whole sample.

| Parameters | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
|---|---|---|---|---|---|---|---|---|---|
| STRUCTURAL PARAMETERS | | | | | | | | | |
| $\alpha$ | 46.7172 (6.0992) | 45.3954 (2.9647) | 3.2672 (18.3025) | 2.7662 (0.0232) | 2.1292 (0.3073) | 5.0486 (0.2162) | -1.0545 (0.5854) | 1.1762 (0.4426) | -0.0574 (0.1888) |
| $\sigma^2$ | 0.5770 (0.0057) | 0.5787 (0.8948) | 0.0000 (0.0000) | 0.0000 (0.0000) | 1.1931 (0.4097) | 0.0006 (0.0037) | 1.8230 (0.2498) | 2.0574 (0.6252) | 4.2494 (0.3059) |
| $\kappa$ | 0.0000 (0.0000) | 0.000 (0.0012) | 0.5252 (4.9997) | 0.5852 (0.0154) | 0.6314 (0.0411) | 0.6718 (0.0142) | 0.1843 (0.0081) | 0.6621 (0.0195) | 0.7309 (0.0146) |
| $\gamma$ | - | 0.0098 (0.0009) | - | - | - | - | - | - | - |
| $\phi$ | - | 3.1416 (0.0509) | - | - | - | - | - | - | - |
| $B_x$ | - | - | 1.0 (3.7954) | 0.9026 (0.0342) | 1.0243 (0.0332) | -1.0647 (0.1989) | - | - | - |
| $B_y$ | - | - | 1.0 (23.6158) | -0.1937 (0.0341) | 0.3711 (0.2333) | 2.8779 (0.1574) | - | - | - |
| $\omega_z$ | - | - | $2\pi$0.0487 (1.8681) | $2\pi$0.0763 (0.0124) | $2\pi$0.0715 (0.0330) | $2\pi$0.0258 (0.0072) | - | - | - |
| Period | - | - | 20.5 years | 13 years | 14 years | 39 years | - | - | - |
| SEASONAL PARAMETERS | | | | | | | | | |
| C2 | | | | | | | | | |
| $A_{x,1}$ | - | - | - | -0.0341 (0.0181) | -3.1482 (1.8135) | -21.1646 (1.1073) | -0.2725 (0.0337) | 0.2171 (0.0929) | -2.3336 (1.7732) |
| $A_{y,1}$ | - | - | - | -0.0893 (0.0158) | 9.9710 (0.7681) | -2.7597 (0.4128) | -0.4517 (0.0417) | -0.1056 (0.0575) | -6.7373 (0.8488) |
| $\omega_{f,1}$ | - | - | - | $2\pi$0.2270 (0.0459) | $2\pi$0.1175 (0.0303) | $2\pi$0.2928 (0.0141) | $2\pi$0.10 (0.0147) | $2\pi$0.1787 (0.0697) | $2\pi$0.1473 (0.0304) |
| Period | - | - | - | 5 years | 8.5 years | 3.5 years | 10 years | 5.5 years | 7 years |
| $A_{x,2}$ | - | - | - | - | 3.7254 (1.7041) | 0.0145 (0.0048) | - | 0.2649 (0.1055) | 13.0661 (2.2216) |
| $A_{y,2}$ | - | - | - | - | -9.8436 (0.8479) | -0.0233 (0.0034) | - | -0.7122 (0.0953) | 16.6759 (4.0166) |
| $\omega_{f,2}$ | - | - | - | - | $2\pi$0.1160 (0.0423) | $2\pi$0.7075 (0.0465) | - | $2\pi$0.10 (0.0331) | $2\pi$0.1647 (0.0184) |
| Period | - | - | - | - | 8.5 years | 1.5 years | - | 10 years | 6 years |
| $A_{x,3}$ | - | - | - | - | - | 21.1372 (1.1082) | - | - | -10.1273 (0.7493) |
| $A_{y,3}$ | - | - | - | - | - | 2.7031 (0.4123) | - | - | -10.6168 (4.6234) |
| $\omega_{f,3}$ | - | - | - | - | - | $2\pi$0.2930 (0.0141) | - | - | $2\pi$0.1704 (0.0285) |
| Period | - | - | - | - | - | 3.5 years | - | - | 6 years |

| Parameters | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | C3 | | | | |
| $A_{x,1}$ | - | - | - | -0.1011 (0.0100) | -0.8144 (0.5527) | 4.9540 (0.2063) | -0.3908 (0.0271) | 0.0920 (0.0755) | -3.9286 (0.2467) |
| $A_{y,1}$ | - | - | - | 0.0033 (0.0151) | 3.3231 (0.5955) | 7.8659 (0.4444) | -0.3923 (0.0437) | -0.6183 (0.0210) | -3.8063 (0.3533) |
| $\omega_{f,1}$ | - | - | - | $2\pi$0.2616 (0.0282) | $2\pi$0.2472 (0.0291) | $2\pi$0.1402 (0.0200) | $2\pi$0.10 (0.0102) | $2\pi$0.10 (0.0218) | $2\pi$0.1342 (0.0129) |
| Period | - | - | - | 4 years | 4 years | 7 years | 10 years | 10 years | 7.5 years |
| $A_{x,2}$ | - | - | - | - | 0.8135 (0.5541) | -4.7483 (0.1924) | - | -0.10 (0.0309) | 4.2508 (0.1781) |
| $A_{y,2}$ | - | - | - | - | -3.3595 (0.5829) | -8.0998 (0.4536) | - | -0.1330 (0.0365) | 2.5631 (0.3664) |
| $\omega$ f,2 | - | - | - | - | $2\pi$0.2461 (0.0451) | $2\pi$0.1392 (0.0198) | - | $2\pi$0.2270 (0.0466) | $2\pi$0.1414 (0.0137) |
| Period | - | - | - | - | 4 years | 7 years | - | 4.5 years | 7 years |
| $A_{x,3}$ | - | - | - | - | - | 0.0008 (0.0148) | - | - | 0.0468 (0.0059) |
| $A_{y,3}$ | - | - | - | - | - | 0.0747 (0.0051) | - | - | -0.0116 ( 0.0132) |
| $\omega_{f,3}$ | - | - | - | - | - | $2\pi$0.3033 (0.0438) | - | - | $2\pi$0.3596 (0.0607) |
| Period | - | - | - | - | - | 3 years | - | - | 3 years |
| | | | | | C4 | | | | |
| $A_{x,1}$ | - | - | - | -0.0628 (0.0260) | 3.1510 (0.2851) | 0.1112 (0.0145) | -0.4650 (0.0191) | -0.1409 (0.0141) | -0.1453 (0.0091) |
| $A_{y,1}$ | - | - | - | 0.0629 (0.0342) | 0.6634 (0.4589) | -0.0312 (0.0315) | -0.5121 (0.0441) | -0.0147 (0.0384) | -0.0025 (0.0227) |
| $\omega_{f,1}$ | - | - | - | $2\pi$0.2761 (0.0236) | $2\pi$0.2207 (0.1191) | $2\pi$0.1399 (0.0576) | $2\pi$0.10 (0.0134) | $2\pi$0.2442 (0.0576) | $2\pi$0.2478 (0.0298) |
| Period | - | - | - | 4 years | 4.5 years | 7 years | 10 years | 4 years | 4 years |
| $A_{x,2}$ | - | - | - | - | -3.2104 (0.2875) | -1.3032 (0.1339) | - | -0.0739 (0.1005) | 0.0378 (0.0763) |
| $A_{y,2}$ | - | - | - | - | -0.5431 (0.4256) | -2.6795 (0.0998) | - | -0.5807 (0.0204) | -0.5884 (0.0082) |
| $\omega_{f,2}$ | - | - | - | - | $2\pi$0.2227 (0.0346) | $2\pi$0.2667 (0.0219) | - | $2\pi$0.1629 (0.0310) | $2\pi$0.1000 (0.0231) |
| Period | - | - | - | - | 4.5 years | 4 years | - | 10 years | 10 years |
| $A_{x,3}$ | - | - | - | - | - | 1.2502 (0.1261) | - | - | -0.0114 (0.0011) |
| $A_{y,3}$ | - | - | - | - | - | 2.7106 (0.1019) | - | - | 0.0053 (0.0016) |
| $\omega_{f,3}$ | - | - | - | - | - | $2\pi$0.2673 (0.0217) | - | - | $2\pi$1.4294 (0.0354) |
| Period | - | - | - | - | - | 4 years | - | - | 1 año |

| Parameters | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | C5 | | | | |
| $A_{x,1}$ | - | - | - | -0.0160 (0.0580) | -0.0417 (0.0138) | 0.9022 (1.2219) | -0.5121 (0.0186) | -0.2062 (0.0257) | -5.8394 (0.8724) |
| $A_{y,1}$ | - | - | - | 0.0902 (0.0095) | 0.0872 (0.0158) | 43.9179 (2.9767) | -0.3103 (0.0331) | -0.0861 (0.0912) | 7.5718 (1.1985) |
| $\omega_{f,1}$ | - | - | - | $2\pi$0.2877 (0.0282) | $2\pi$0.2786 (0.0455) | $2\pi$0.2000 (0.0193) | $2\pi$0.10 (0.0086) | $2\pi$0.2277 (0.0714) | $2\pi$0.1721 (0.0080) |
| Period | - | - | - | 3.5 years | 3.5 years | 5 years | 10 years | 4.5 years | 6 years |
| $A_{x,2}$ | - | - | - | - | -0.1617 (0.0544) | -3.8639 (0.4296) | - | -0.5195 (0.1228) | 4.7467 (0.8774) |
| $A_{y,2}$ | - | - | - | - | -0.0074 (0.0659) | -21.8634 (1.5149) | - | -0.2867 (0.1569) | -7.9246 (1.1865) |
| $\omega_{f,2}$ | - | - | - | - | $2\pi$0.10 (0.0403) | $2\pi$0.2039 (0.0184) | - | $2\pi$0.1270 (0.0587) | $2\pi$0.1757 (0.0080) |
| Period | - | - | - | - | 10 years | 5 years | - | 8 years | 6 years |
| $A_{x,3}$ | - | - | - | - | - | 2.6764 (0.8056) | - | - | 0.0011 (0.0020) |
| $A_{y,3}$ | - | - | - | - | - | -21.5944 (1.4597) | - | - | 0.0114 (0.0009) |
| $\omega_{f,3}$ | - | - | - | - | - | $2\pi$0.1964 (0.0202) | - | - | $2\pi$1.4448 (0.0390) |
| Period | - | - | - | - | - | 5 years | - | - | 1 año |
| | | | | | C6 | | | | |
| $A_{x,1}$ | - | - | - | 0.0215 (0.0126) | 0.0234 (0.0315) | 0.5844 (0.2212) | -0.5033 (0.0112) | 0.0055 (0.0230) | -0.1831 (0.0523) |
| $A_{y,1}$ | - | - | - | 0.0816 (0.0210) | 0.0893 (0.0102) | -1.8668 (0.1094) | -0.2266 (0.0611) | 0.1055 (0.0101) | -0.4224 (0.0337) |
| $\omega_{f,1}$ | - | - | - | $2\pi$0.2975 (0.0274) | $2\pi$0.2972 (0.0459) | $2\pi$0.3346 (0.0200) | $2\pi$0.10 (0.0184) | $2\pi$0.2885 (0.0430) | $2\pi$0.1015 (0.0251) |
| Period | - | - | - | 3.5 years | 3.5 years | 3 years | 10 years | 4 years | 10 years |
| $A_{x,2}$ | - | - | - | - | -0.2252 (0.0357) | 0.2477 (0.1625) | - | -0.1731 (0.0599) | 0.0381 (0.0153) |
| $A_{y,2}$ | - | - | - | - | -0.0305 (0.0460) | 1.4975 (0.1126) | - | -0.4760 (0.0401) | 0.1014 (0.0110) |
| $\omega_{f,2}$ | - | - | - | - | $2\pi$0.10 (0.0304) | $2\pi$0.3476 (0.0205) | - | $2\pi$0.10 (0.0210) | $2\pi$0.2973 (0.0342) |
| Period | - | - | - | - | 10 years | 3 years | - | 10 years | 3.5 years |
| $A_{x,3}$ | - | - | - | - | - | -0.4568 (0.0293) | - | - | 0.0086 (0.0029) |
| $A_{y,3}$ | - | - | - | - | - | -0.2293 (0.0597) | - | - | 0.0078 (0.0030) |
| $\omega_{f,3}$ | - | - | - | - | - | $2\pi$0.2810 (0.0204) | - | - | $2\pi$1.4546 (0.0636) |
| Period | - | - | - | - | - | 3.5 years | - | - | 1 año |
| $\sum_t \min SCR(\hat{\theta}_t) = \sum_t \hat{u}_t$ | 122.26 | 121.24 | 67.21 | 59.7646 | 58.4287 | 57.8309 | 64.2147 | 58.3276 | 57.0128 |
| $(\frac{1}{n}\sum_{t=1}^{n}\hat{u}_t)^{1/2}$ | 0.1280 | 0.1274 | 0.0949 | 0.0895 | 0.0885 | 0.0880 | 0.0927 | 0.0884 | 0.0874 |
| $\frac{1}{n}\sum_{t=1}^{n}\lvert\hat{u}_t\rvert$ | 0.0887 | 0.0886 | 0.0561 | 0.0499 | 0.0495 | 0.0495 | 0.0499 | 0.0494 | 0.0490 |

**Table 3**: Parameters of the in-sample estimations for the nine one-factor models, periods, and goodness of fit measures for the whole sample period. In parenthesis we show the standard deviations of the estimations.

SCR, seasonality, and long-term oscillations. Sample period: first available data - 2018.

| $\sum_t \min SCR(\hat{\theta}_t)$ | C2 | C3 | C4 | C5 | C6 |
|---|---|---|---|---|---|
| **Model 1** | 8.15 | 18.92 | 17.51 | 18.89 | 29.46 |
| **Model 2** | 8.08 | 18.72 | 17.40 | 18.83 | 29.42 |
| Seasonality (years) | 1 | 1 | 1 | 1 | 1 |
| Improvement over model 1 (%) | 0.83 | 1.08 | 0.63 | 0.33 | 0.14 |
| **Model 3** | 7.002 | 14.70 | 10.00 | 7.33 | 11.34 |
| Long-term swing (years) | 20.5 | 20.5 | 20.5 | 20.5 | 20.5 |
| Improvement over model 2 (%) | 13.39 | 21.42 | 42.53 | 61.08 | 61.47 |
| **Model 4** | 6.95 | 13.85 | 8.77 | 5.59 | 8.79 |
| Long-term swing (years) | 13 | 13 | 13 | 13 | 13 |
| Seasonality (years) | 4.5 | 4 | 4 | 3.5 | 3.5 |
| Improvement over model 3 (%) | 0.74 | 5.78 | 12.30 | 23.74 | 22.49 |
| **Model 5** | 6.69 | 13.63 | 8.80 | 5.62 | 8.46 |
| Long-term swing (years) | 14 | 14 | 14 | 14 | 14 |
| Seasonality (years) | 8.5 | 4 | 4.5 | 3.5 | 3.5 |
| Seasonality (years) | 8.5 | 4 | 4.5 | 10 | 10 |
| Improvement over model 4 (%) | 3.74 | 1.59 | -0.34 | -0.54 | 3.75 |
| **Model 6** | 6.67 | 13.41 | 8.72 | 5.74 | 8.09 |
| Long-term swing (years) | 13.5 | 13.5 | 13.5 | 13.5 | 13.5 |
| Seasonality (years) | 3.5 | 7 | 7 | 5 | 3 |
| Seasonality (years) | 1.5 | 7 | 4 | 5 | 3 |
| Seasonality (years) | 3.5 | 3 | 4 | 5 | 3.5 |
| Improvement over model 5 (%) | 0.30 | 1.61 | 0.91 | 2.14 | 4.37 |
| **Model 7** | 6.92 | 14.84 | 9.53 | 6.04 | 9.53 |
| Seasonality (years) | 10 | 10 | 10 | 10 | 10 |
| Improvement over model 2 (%) | 14.36 | 20.73 | 45.23 | 67.93 | 67.61 |
| **Model 8** | 6.62 | 13.57 | 8.74 | 5.74 | 8.56 |
| Seasonality (years) | 5.5 | 10 | 5 | 4.5 | 3.5 |
| Seasonality (years) | 10 | 5 | 10 | 8 | 10 |
| Improvement over model 7 (%) | 4.34 | 8.56 | 8.29 | 4.97 | 10.18 |
| **Model 9** | 6.51 | 13.27 | 8.61 | 5.80 | 8.46 |
| Seasonality (years) | 7 | 7.5 | 4 | 6 | 10 |
| Seasonality (years) | 6 | 7 | 10 | 6 | 3.5 |
| Seasonality (years) | 6 | 3 | 1 | 1 | 1 |
| Improvement over model 8 (%) | 1.66 | 2.21 | 1.49 | -1.05 | 1.17 |

**Table 4**: Sum of squared pricing errors, seasonal periods, long-term swing in the mean-reversion level, and statistical improvement of each model. Sample: first available data until 2018.

## Parametric estimations, since available data until 2013.

| Parameters | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
|---|---|---|---|---|---|---|---|---|---|
| STRUCTURAL PARAMETERS | | | | | | | | | |
| $\alpha$ | 6.0620 (0.2158) | 5.7335 (0.1734) | 3.0212 (98.7283) | -0.5866 (0.3431) | 3.9163 (0.0698) | 9.3737 (0.1526) | 0.23141 (0.1828) | 1.7790 (0.0296) | 1.8559 (0.0239) |
| $\sigma^2$ | 0.0000 (0.0000) | 0.0000 (0.0000) | 0.0000 (0.0000) | 6.8079 (0.4039) | 6.9084 (0.3060) | 7.2207 (0.2662) | 7.8655 (0.4547) | 5.0078 (0.2306) | 7.3708 (0.2201) |
| $\kappa$ | 0.1663 (0.0102) | 0.1871 (0.0086) | 0.8251 (16.7225) | 2.0768 (0.0360) | 3.3624 (0.0696) | 3.4368 (0.0606) | 1.2271 (0.0214) | 2.6670 (0.0398) | 3.3863 (0.0552) |
| $\gamma$ | - | 0.0312 (0.0007) | - | - | - | - | - | - | - |
| $\phi$ | - | 0.2046 (0.0041) | - | - | - | - | - | - | - |
| $B_x$ | - | - | 0.7297 (219.8136) | 0.9948 (0.3417) | -6.0768 (0.1347) | 1.1014 (0.0425) | - | - | - |
| $B_y$ | - | - | -0.0093 (145.8114) | -2.1419 (0.2154) | 5.1709 (0.1056) | 1.7244 (0.1202) | - | - | - |
| $\omega_z$ | - | - | $2\pi$0.10 (186.7926) | $2\pi$0.10 (0.0496) | $2\pi$0.0582 (0.0021) | $2\pi$0.0522 (0.0064) | - | - | - |
| Period | - | - | 10 years | 10 years | 10 years | 17 years | - | - | - |
| SEASONAL PARAMETERS | | | | | | | | | |
| C2 | | | | | | | | | |
| $A_{x,1}$ | - | - | - | 0.4686 (0.0237) | -0.0727 (0.0024) | 2.3601 (0.2590) | 0.5857 (0.0153) | 14.4101 (0.6657) | -1.6247 (0.0581) |
| $A_{y,1}$ | - | - | - | -0.3636 (0.0253) | -0.0032 (0.0058) | 1.3521 (0.1210) | 0.3429 (0.0578) | -4.2427 (1.8977) | -0.4881 (0.1704) |
| $\omega_{f,1}$ | - | - | - | $2\pi$0.2245 (0.0171) | $2\pi$0.8649 (0.0429) | $2\pi$0.5024 (0.0184) | $2\pi$0.10 (0.0361) | $2\pi$0.2244 (0.0245) | $2\pi$0.2542 (0.0303) |
| Period | - | - | - | 4.5 years | 1 año | 2 years | 10 years | 4.5 years | 4 years |
| $A_{x,2}$ | - | - | - | - | 0.2662 (0.1637) | -0.1674 (0.0097) | - | -14.4283 (0.6657) | 1.5856 (0.0740) |
| $A_{y,2}$ | - | - | - | - | -2.0712 (0.0697) | -0.2328 (0.0327) | - | 3.8628 (1.8979) | 0.5516 (0.1655) |
| $\omega_{f,2}$ | - | - | - | - | $2\pi$0.1729 (0.0222) | $2\pi$0.7508 (0.0494) | - | $2\pi$0.2243 (0.0246) | $2\pi$0.2332 (0.0300) |
| Period | - | - | - | - | 6 years | 1.5 years | - | 4.5 years | 4 years |
| $A_{x,3}$ | - | - | - | - | - | -2.5456 (0.2874) | - | - | -0.0665 (0.0026) |
| $A_{y,3}$ | - | - | - | - | - | -0.0817 (0.0827) | - | - | 0.0189 (0.0051) |
| $\omega_{f,3}$ | - | - | - | - | - | $2\pi$0.5441 (0.0165) | - | - | $2\pi$0.8928 (0.0416) |
| Period | - | - | - | - | - | 2 years | - | - | 1 año |

| Parameters | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | C3 | | | | |
| $A_{x,1}$ | - | - | - | 0.0059 (0.0206) | 0.5384 (0.1674) | 0.0961 (0.0165) | 0.5661 (0.0907) | 8.1621 (0.1650) | -2.0323 (0.1795) |
| $A_{y,1}$ | - | - | - | -0.3946 (0.0184) | -2.2191 (0.0619) | -0.1842 (0.0346) | -0.1633 (0.0850) | 19.2284 (0.6285) | 4.5440 (0.4813) |
| $\omega_{f,1}$ | - | - | - | $2\pi$0.2736 (0.0211) | $2\pi$0.1626 (0.0196) | $2\pi$0.4176 (0.0221) | $2\pi$0.2145 (0.0816) | $2\pi$0.3949 (0.0161) | $2\pi$0.4448 (0.0327) |
| Period | - | - | - | 4 years | 6 years | 2.5 years | 8 years | 2.5 years | 2 years |
| $A_{x,2}$ | - | - | - | - | -0.0749 (0.0033) | -0.7442 (0.0415) | - | -8.7701 (0.1634) | 0.9065 (0.2272) |
| $A_{y,2}$ | - | - | - | - | 0.0313 (0.0057) | -0.0840 (0.0568) | - | -18.6820 (0.6356) | -4.6988 (0.4403) |
| $\omega$ f,2 | - | - | - | - | $2\pi$0.8747 (0.0379) | $2\pi$0.2869 (0.0357) | - | $2\pi$0.3968 (0.0161) | $2\pi$0.4631 (0.0289) |
| Period | - | - | - | - | 1 año | 3.5 years | - | 2.5 years | 2 years |
| $A_{x,3}$ | - | - | - | - | - | -0.0965 (0.029) | - | - | -0.0337 (0.0162) |
| $Ay, 3$ | - | - | - | - | - | -0.0509 (0.0101) | - | - | -0.2298 (0.0201) |
| $\omega_{f,3}$ | - | - | - | - | - | $2\pi$0.8072 (0.0466) | - | - | $2\pi$0.7322 (0.0407) |
| Period | - | - | - | - | - | 1.5 years | - | - | 1.5 years |
| | | | | | C4 | | | | |
| $A_{x,1}$ | - | - | - | -0.2399 (0.0114) | 0.3621 (0.1581) | 20.8899 (0.7646) | -0.2264 (0.0110) | 0.8159 (0.0329) | -0.2213 (0.0139) |
| $A_{y,1}$ | - | - | - | 0.0065 (0.0185) | -2.3290 (0.0643) | 1.0828 (0.1518) | 0.6999 (0.0281) | -0.2218 (0.0366) | -0.0464 (0.0162) |
| $\omega_{f,1}$ | - | - | - | $2\pi$0.3678 (0.0296) | $2\pi$0.1639 (0.0175) | $2\pi$0.2867 (0.0199) | $2\pi$0.2462 (0.0474) | $2\pi$0.1014 (0.0186) | $2\pi$0.3177 (0.0327) |
| Period | - | - | - | 3 years | 6 years | 3.5 years | 4 years | 10 years | 3 years |
| $A_{x,2}$ | - | - | - | - | -0.0402 (0.0589) | -21.5728 (0.7637) | - | -0.0695 (0.0036) | -0.0563 (0.0115) |
| $A_{y,2}$ | - | - | - | - | 0.0589 (0.0032) | -1.4009 (0.1432) | - | -0.0447 (0.0051) | -0.1959 (0.0162) |
| $\omega_{f,2}$ | - | - | - | - | $2\pi$0.8766 (0.0395) | $2\pi$0.2851 (0.0186) | - | $2\pi$0.7463 (0.0310) | $2\pi$0.2034 (0.0213) |
| Period | - | - | - | - | 1 año | 3.5 years | - | 1.5 years | 5 years |
| $A_{x,3}$ | - | - | - | - | - | -0.0709 (0.0063) | - | - | -0.0086 (0.0063) |
| $A_{y,3}$ | - | - | - | - | - | 0.0397 (0.0074) | - | - | 0.0627 (0.0014) |
| $\omega_{f,3}$ | - | - | - | - | - | $2\pi$0.8459 (0.0555) | - | - | $2\pi$0.9111 (0.0506) |
| Period | - | - | - | - | - | 1 año | - | - | 1 año |

| Parameters | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | C5 | | | | |
| $A_{x,1}$ | - | - | - | 0.1331 (0.0074) | 10.4344 (0.1880) | 20.1130 (0.1466) | 0.6999 (0.1213) | 1.5904 (0.0618) | 0.5213 (0.0102) |
| $A_{y,1}$ | - | - | - | -0.0520 (0.0136) | 11.9826 (0.2923) | -10.8276 (0.3661) | -0.3858 (0.0755) | -0.2047 (0.0748) | -0.1495 (0.0274) |
| $\omega_{f,1}$ | - | - | - | $2\pi$0.5753 (0.0386) | $2\pi$0.3822 (0.0087) | $2\pi$0.4696 (0.0196) | $2\pi$0.10 (0.0493) | $2\pi$0.4146 (0.0189) | $2\pi$0.4786 (0.0257) |
| Period | - | - | - | 2 years | 2.5 years | 2 years | 10 years | 2.5 years | 2 years |
| $A_{x,2}$ | - | - | - | - | -12.0126 (0.2186) | -19.3101 (0.2432) | - | -0.6532 (0.0741) | 0.3258 (0.0074) |
| $A_{y,2}$ | - | - | - | - | -7.9467 (0.2230) | 12.3856 (0.1674) | - | 1.2013 (0.0498) | 0.0804 (0.0189) |
| $\omega_{f,2}$ | - | - | - | - | $2\pi$0.3974 (0.0089) | $2\pi$0.4759 (0.0231) | - | $2\pi$0.4652 (0.0187) | $2\pi$0.6208 (0.0290) |
| Period | - | - | - | - | 2.5 years | 2 years | - | 2 years | 1.5 years |
| $A_{x,3}$ | - | - | - | - | - | 0.8929 (0.3621) | - | - | 0.0271 (0.0015) |
| $A_{y,3}$ | - | - | - | - | - | -0.2606 (0.2314) | - | - | 0.0029 (0.0065) |
| $\omega_{f,3}$ | - | - | - | - | - | $2\pi$0.5033 (0.0196) | - | - | $2\pi$1.5269 (0.0915) |
| Period | - | - | - | - | - | 2 years | - | - | 1 año |
| | | | | | C6 | | | | |
| $A_{x,1}$ | - | - | - | 0.2384 (0.2921) | -14.0722 (0.3386) | -7.9319 (0.7617) | 0.3214 (0.6993) | 0.0346 (0.0169) | 4.4377 (0.1740) |
| $A_{y,1}$ | - | - | - | -0.3562 (0.1160) | 0.7565 (0.0.1857) | -29.6918 (0.8623) | -0.2158 (0.3670) | -0.0214 (0.0224) | 1.9128 (0.0470) |
| $\omega_{f,1}$ | - | - | - | $2\pi$0.2843 (0.2345) | $2\pi$0.5769 (0.0104) | $2\pi$0.4693 (0.0164) | $2\pi$0.1001 (0.5373) | $2\pi$1.0404 (0.1816) | $2\pi$0.6923 (0.0116) |
| Period | - | - | - | 3.5 years | 1 año | 2 years | 10 years | 1 año | 1.5 years |
| $A_{x,2}$ | - | - | - | - | 11.1213 (0.1925) | 8.7426 (0.4560) | - | 0.0288 (0.0480) | 1.2792 (0.0190) |
| $A_{y,2}$ | - | - | - | - | -6.5650 (0.3163) | 11.9233 (0.8693) | - | -0.0516 (0.0259) | -3.6162 (0.1550) |
| $\omega_{f,2}$ | - | - | - | - | $2\pi$0.6017 (0.0112) | $2\pi$0.4645 (0.0161) | - | $2\pi$1.5764 (0.2810) | $2\pi$0.6115 (0.0103) |
| Period | - | - | - | - | 1.5 years | 2 years | - | 1 año | 1.5 years |
| $A_{x,3}$ | - | - | - | - | - | 3.5817 (0.2012) | - | - | 1.0357 (0.0166) |
| $A_{y,3}$ | - | - | - | - | - | 17.5365(0.0884) | - | - | -0.5751 (0.0563) |
| $\omega_{f,3}$ | - | - | - | - | - | $2\pi$0.4843 (0.0136) | - | - | $2\pi$0.9078 (0.0164) |
| Period | - | - | - | - | - | 2 years | - | - | 1 año |
| $\sum_t \min SCR(\hat{\theta}_t) = \sum_t \hat{u}_t$ | 24.2634 | 21.8772 | 21.8082 | 18.1176 | 15.1486 | 15.1911 | 19.5338 | 16.9416 | 14.8985 |
| $(\frac{1}{n}\sum_{t=1}^{n} \hat{u}_t)^{1/2}$ | 0.1607 | 0.1526 | 0.1523 | 0.1388 | 0.1264 | 0.1271 | 0.1442 | 0.1342 | 0.1259 |
| $\frac{1}{n}\sum_{t=1}^{n} \lvert\hat{u}_t\rvert$ | 0.0656 | 0.0652 | 0.0608 | 0.0574 | 0.0578 | 0.0582 | 0.0593 | 0.0565 | 0.0566 |

**Table 5**: Parameters of the in-sample estimations for the nine one-factor models, periods, and goodness of fit measures. The sample is from the first available data until 2013. In parenthesis we show the standard deviations of the estimations.

SCR, seasonality, and long-term oscillations. Sample period: first available data - 2012.

| $\sum_t \min SCR(\hat{\theta}_t)$ | C2 | C3 | C4 | C5 | C6 |
|---|---|---|---|---|---|
| **Model 1** | 1.13 | 2.20 | 1.36 | 1.04 | 1.25 |
| **Model 2** | 1.08 | 1.99 | 1.24 | 0.99 | 1.20 |
| Seasonality (years) | 1 | 1 | 1 | 1 | 1 |
| Improvement over model 1 (%) | 4.9 | 9.46 | 8.53 | 5.26 | 4.05 |
| **Model 3** | 1.06 | 1.82 | 1.03 | 0.74 | 0.94 |
| Long-term swing (years) | 10 | 10 | 10 | 10 | 10 |
| Improvement over model 2 (%) | 1.61 | 8.51 | 17.49 | 25.11 | 22.29 |
| **Model 4** | 0.88 | 1.48 | 0.93 | 0.72 | 0.74 |
| Long-term swing (years) | 10 | 10 | 10 | 10 | 10 |
| Seasonality (years) | 4.5 | 4 | 3 | 2 | 3.5 |
| Improvement over model 3 (%) | 16.75 | 18.51 | 6.54 | 3.17 | 21.11 |
| **Model 5** | 0.84 | 1.30 | 0.99 | 0.84 | 0.65 |
| Long-term swing (years) | 10 | 10 | 10 | 10 | 10 |
| Seasonality (years) | 1 | 6 | 6 | 2.5 | 2 |
| Seasonality (years) | 6 | 1 | 1 | 2.5 | 1 |
| Improvement over model 4 (%) | 4.54 | 12.34 | -6.29 | -16.70 | 11.29 |
| **Model 6** | 0.84 | 1.26 | 0.98 | 0.87 | 0.69 |
| Long-term swing (years) | 17 | 17 | 17 | 17 | 17 |
| Seasonality (years) | 2 | 2.5 | 3.5 | 2 | 2 |
| Seasonality (years) | 1.5 | 3.5 | 3.5 | 2 | 2 |
| Seasonality (years) | 2 | 1.5 | 1 | 2 | 2 |
| Improvement over model 5 (%) | 0.02 | 2.88 | 0.84 | -3.93 | -6.13 |
| **Model 7** | 0.93 | 1.67 | 1.05 | 0.77 | 0.79 |
| Seasonality (years) | 10 | 8 | 4 | 10 | 10 |
| Improvement over model 2 (%) | 13.13 | 16.28 | 15.77 | 21.87 | 34.66 |
| **Model 8** | 0.89 | 1.41 | 0.89 | 0.75 | 0.64 |
| Seasonality (years) | 4.5 | 2.5 | 10 | 2.5 | 1 |
| Seasonality (years) | 4.5 | 2.5 | 1.5 | 2 | 1 |
| Improvement over model 7 (%) | 4.81 | 15.51 | 15.33 | 3.36 | 19.09 |
| **Model 9** | 0.85 | 1.25 | 1.01 | 0.72 | 0.64 |
| Seasonality (years) | 4 | 2 | 3 | 2 | 1.5 |
| Seasonality (years) | 4 | 2 | 5 | 1.5 | 1.5 |
| Seasonality (years) | 1 | 1.5 | 1 | 1 | 1.5 |
| Improvement over model 8 (%) | 3.85 | 10.85 | -13.82 | 2.93 | 0.31 |

**Table 6**: Sum of squared pricing errors, seasonal periods, long-term swing in the mean reversion level, and statistical improvement of each model. Sample: first available data until 2012.

Parametric estimations for the period 2013-2018.

| Parameters | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
|---|---|---|---|---|---|---|---|---|---|
| STRUCTURAL PARAMETERS | | | | | | | | | |
| $\alpha$ | 2.1024 (0.0063) | 2.0919 (0.0566) | 2.7194 (0.0360) | 2.0465 (0.7347) | 2.9701 (0.2013) | 11.1717 (0.1213) | 2.1345 (0.0078) | 0.7396 (0.1845) | 1.2160 (0.1501) |
| $\sigma^2$ | 0.0000 (0.0000) | 0.0000 (0.0000) | 0.0000 (0.0000) | 2.6722 (0.5054) | 1.7800 (0.1989) | 1.0584 (0.1069) | 0.0000 (0.0000) | 2.8723 (0.3173) | 2.5383 (0.2796) |
| $\kappa$ | 0.6061 (0.0101) | 0.6276 (0.0824) | 0.2834 (0.0084) | 0.7233 (0.0154) | 0.7746 (0.0174) | 0.7600 (0.0143) | 0.5509 (0.0601) | 0.7553 (0.0142) | 0.8299 (0.0497) |
| $\gamma$ | - | -0.0048 (0.0007) | - | - | - | - | - | - | - |
| $\phi$ | - | 1.0962 (0.0241) | - | - | - | - | - | - | - |
| $B_x$ | - | - | -0.0668 (0.0760) | 3.1648 (0.9031) | 2.1618 (0.1353) | -2.3127 (0.1557) | - | - | - |
| $B_y$ | - | - | -1.0 (0.0398) | 1.7277 (1.7081) | 1.5273 (0.1846) | 10.0570 (0.1334) | - | - | - |
| $\omega_z$ | - | - | $2\pi$0.10 (0.0119) | $2\pi$0.0856 (0.1020) | $2\pi$0.0737 (0.0062) | $2\pi$0.0370 (0.0028) | - | - | - |
| Period | - | - | 10 years | 12 years | 13.5 years | 27 years | - | - | |
| SEASONAL PARAMETERS | | | | | | | | | |
| C2 | | | | | | | | | |
| $A_{x,1}$ | - | - | - | 0.5837 (0.1920) | 24.4853 (0.1721) | 2.1935 (0.4392) | 0.0822 (0.0299) | -16.2817 (1.0068) | 9.9040 (1.1590) |
| $A_{y,1}$ | - | - | - | 1.1918 (0.4895) | 1.3200 (2.5632) | -5.2457 (0.5131) | -0.1473 (0.0494) | 26.3427 (1.2936) | -21.1874 (0.3571) |
| $\omega_{f,1}$ | - | - | - | $2\pi$0.1408 (0.0398) | $2\pi$0.2475 (0.0305) | $2\pi$0.2930 (0.0212) | $2\pi$0.2162 (0.0703) | $2\pi$0.1209 (0.0048) | $2\pi$0.1896 (0.0119) |
| Period | - | - | - | 7 years | 4 years | 3.5 years | 4.5 years | 8 years | 5 years |
| $A_{x,2}$ | - | - | - | - | -24.8286 (0.2350) | 11.6159 (0.6748) | - | 18.1015 (1.0364) | -13.8954 (1.0497) |
| $A_{y,2}$ | - | - | - | - | -2.1523 (2.6005) | 15.5812 (0.9319) | - | -25.6476 (1.2769) | 16.1317 (0.4269) |
| $\omega_{f,2}$ | - | - | - | - | $2\pi$0.2467 (0.0149) | $2\pi$0.2908 (0.0219) | - | $2\pi$0.1191 (0.0045) | $2\pi$0.1820 (0.0125) |
| Period | - | - | - | - | 4 years | 3.5 years | - | 8 years | 5.5 years |
| $A_{x,3}$ | - | - | - | - | - | -14.5431 (0.4272) | - | - | 2.0090 (0.1254) |
| $A_{y,3}$ | - | - | - | - | - | -9.7698 (0.4871) | - | - | 1.7248 (0.1096) |
| $\omega_{f,3}$ | - | - | - | - | - | $2\pi$0.2917 (0.0226) | - | - | $2\pi$0.2244 (0.0203) |
| Period | - | - | - | - | - | 3.5 years | - | - | 4.5 years |

| Parameters | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | C3 | | | | |
| $A_{x,1}$ | - | - | - | 0.5187 (0.2103) | -36.4487 (1.1556) | -19.9361 (0.4335) | 0.0818 (0.0205) | -19.2686 (0.9964) | -26.0575 (1.6385) |
| $A_{y,1}$ | - | - | - | 1.3027 (0.4964) | -6.6253 (3.0065) | 0.6519 (0.1372) | -0.1867 (0.1355) | 8.8366 (0.2377) | 15.3824 (1.2162) |
| $\omega_{f,1}$ | - | - | - | $2\pi$0.1386 (0.0374) | $2\pi$0.2411 (0.0089) | $2\pi$0.3235 (0.0055) | $2\pi$0.2182 (0.0588) | $2\pi$0.1121 (0.0033) | $2\pi$0.1185 (0.0099) |
| Period | - | - | - | 7 years | 4 years | 3 years | 4.5 years | 9 years | 8.5 years |
| $A_{x,2}$ | - | - | - | - | 36.1834 (1.1509) | 19.1923 (0.3961) | - | 20.7021 (1.0037) | 28.4936 (1.5290) |
| $A_{y,2}$ | - | - | - | - | 5.5568 (3.0099) | 2.3764 (0.1526) | - | -6.4771 (0.1802) | -8.2194 (1.3070) |
| $\omega_{f,2}$ | - | - | - | - | $2\pi$0.2418 (0.0075) | $2\pi$0.3195 (0.0054) | - | $2\pi$0.1086 (0.0027) | $2\pi$0.118 (0.0096) |
| Period | - | - | - | - | 4 years | 3 years | - | 9 years | 9 years |
| $A_{x,3}$ | - | - | - | - | - | -0.4267 (0.0323) | - | - | -1.0141 (0.1214) |
| $A_{y,3}$ | - | - | - | - | - | -0.8069 (0.0363) | - | - | -1.1894 (0.1688) |
| $\omega_{f,3}$ | - | - | - | - | - | $2\pi$0.3756 (0.0102) | - | - | $2\pi$0.1647 (0.0178) |
| Period | - | - | - | - | - | 2.5 years | - | - | 6 years |
| | | | | | C4 | | | | |
| $A_{x,1}$ | - | - | - | 0.2683 (0.2457) | 6.8861 (0.5925) | 4.0336 (0.1149) | 0.1744 (0.0110) | 1.3028 (0.1847) | 16.1787 (1.6827) |
| $A_{y,1}$ | - | - | - | 1.4576 (0.4713) | 8.3156 (0.9931) | 12.3560 (0.0947) | 0.2052 (0.1419) | 1.3076 (0.0288) | 27.6922 (2.0443) |
| $\omega_{f,1}$ | - | - | - | $2\pi$0.1333 (0.0309) | $2\pi$0.2226 (0.0147) | $2\pi$0.2864 (0.0058) | $2\pi$0.2023 (0.0249) | $2\pi$0.1502 (0.0075) | $2\pi$0.1398 (0.0216) |
| Period | - | - | - | 7 years | 4.5 years | 3.5 years | 5 years | 7 years | 7 years |
| $A_{x,2}$ | - | - | - | - | -6.4906 (0.6108) | -16.5140 (0.2285) | - | -0.3434 (0.2789) | -14.9998 (1.7860) |
| $A_{y,2}$ | - | - | - | - | -9.2355 (0.9613) | -19.4722 (0.2143) | - | -2.0313 (0.0893) | -28.7678 (2.0816) |
| $\omega_{f,2}$ | - | - | - | - | $2\pi$0.2207 (0.0122) | $2\pi$0.2952 (0.0068) | - | $2\pi$0.1306 (0.0175) | $2\pi$0.1383 (0.0218) |
| Period | - | - | - | - | 4.5 years | 3.5 years | - | 7.5 years | 7 years |
| $A_{x,3}$ | - | - | - | - | - | 10.8037 (0.2816) | - | - | 0.0063 (0.0025) |
| $A_{y,3}$ | - | - | - | - | - | 6.9909 (0.27865) | - | - | 0.0054 (0.0076) |
| $\omega_{f,3}$ | - | - | - | - | - | $2\pi$0.3031 (0.0069) | - | - | $2\pi$1.4830 (0.0629) |
| Period | - | - | - | - | - | 3 years | - | - | 1 año |

| Parameters | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 | Model 9 |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | C5 | | | | |
| $A_{x,1}$ | - | - | - | 0.0881 (0.2804) | -0.0129 (0.0042) | 0.0558 (0.0209) | 0.2052 (0.0063) | 6.7533 (0.4439) | 36.6138 (0.6541) |
| $A_{y,1}$ | - | - | - | 1.5548 (0.4494) | 0.0024 (0.0045) | -0.4175 (0.0116) | -0.0540 (0.0462) | -1.3413 (0.0.3134) | 12.6518 (2.5441) |
| $\omega_{f,1}$ | - | - | - | $2\pi$0.1299 (0.0338) | $2\pi$0.7366 (0.0651) | $2\pi$0.1979 (0.0084) | $2\pi$0.1938 (0.0621) | $2\pi$0.1639 (0.0081) | $2\pi$0.1520 (0.0219) |
| Period | - | - | - | 7.5 years | 1.5 years | 5 years | 5 years | 6 years | 6.5 years |
| $A_{x,2}$ | - | - | - | - | 0.5798 (0.0307) | 4.9736 (0.0875) | - | -7.0598 (0.4111) | -36.4593 (0.6516) |
| $A_{y,2}$ | - | - | - | - | -0.0320 (0.0325) | 1.7154 (0.0634) | - | 0.4823 (0.3448) | -14.0639 (2.4403) |
| $\omega_{f,2}$ | - | - | - | - | $2\pi$0.1678 (0.0116) | $2\pi$0.4628 (0.0168) | - | $2\pi$0.1597 (0.0073) | $2\pi$0.1509 (0.0209) |
| Period | - | - | - | - | 6 years | 2 years | - | 6 years | 6.5 years |
| $A_{x,3}$ | - | - | - | - | - | -4.9573 (0.0886) | - | - | -0.0023 (0.0093) |
| $A_{y,3}$ | - | - | - | - | - | -1.8226 (0.0618) | - | - | -0.0100 (0.0014) |
| $\omega_{f,3}$ | - | - | - | - | - | $2\pi$0.4623 (0.0167) | - | - | $2\pi$1.5254 (0.0527) |
| Period | - | - | - | - | - | 2 years | - | - | 1 año |
| | | | | | C6 | | | | |
| $A_{x,1}$ | - | - | - | -0.0351 (0.2888) | -7.6119 (2.6628) | -25.9863 (0.4326) | 0.2182 (0.0056) | -2.6361 (0.2078) | -4.7098 (0.1766) |
| $A_{y,1}$ | - | - | - | 1.6271 (0.4729) | -14.7730 (0.1025) | 4.9824 (0.3952) | -0.0602 (0.0507) | 0.7075 (0.0979) | -1.1071 (0.3393) |
| $\omega_{f,1}$ | - | - | - | $2\pi$0.1276 (0.0283) | $2\pi$0.1615 (0.0098) | $2\pi$0.2523 (0.0133) | $2\pi$0.1929 (0.0492) | $2\pi$0.1625 (0.0045) | $2\pi$0.1521 (0.0164) |
| Period | - | - | - | 7.5 years | 6 years | 4 years | 5 years | 6 years | 6.5 years |
| $A_{x,2}$ | - | - | - | - | 8.2707 (2.6590) | 27.7239 (0.4595) | - | 2.2381 (0.1880) | 4.5605 (0.2079) |
| $A_{y,2}$ | - | - | - | - | 14.8580 (0.1139) | -4.6863 (0.3952) | - | -1.3165 (0.1044) | -0.1282 (0.2376) |
| $\omega_{f,2}$ | - | - | - | - | $2\pi$0.1615 (0.0081) | $2\pi$0.2523 (0.0130) | - | $2\pi$0.1719 (0.0041) | $2\pi$0.1602 (0.0147) |
| Period | - | - | - | - | 6 years | 4 years | - | 6 years | 6 years |
| $A_{x,3}$ | - | - | - | - | - | -1.7794 (0.0462) | - | - | -0.0079 (0.0041) |
| $A_{y,3}$ | - | - | - | - | - | -1.1611 (0.0561) | - | - | -0.0118 (0.0024) |
| $\omega_{f,3}$ | - | - | - | - | - | $2\pi$0.2282 (0.0085) | - | - | $2\pi$1.5250 (0.0491) |
| Period | - | - | - | - | - | 4.5 years | - | - | 1 año |
| $\sum_t \min SCR(\hat{\theta}_t) = \sum_t \hat{u}_t$ | 57.4106 | 57.2739 | 39.7311 | 34.5377 | 34.1768 | 34.1694 | 35.1739 | 34.2873 | 33.753 |
| $(\frac{1}{n}\sum_{t=1}^{n} \hat{u}_t)^{1/2}$ | 0.0938 | 0.0937 | 0.0780 | 0.0727 | 0.0723 | 0.0723 | 0.0734 | 0.0725 | 0.0719 |
| $\frac{1}{n}\sum_{t=1}^{n} \lvert\hat{u}_t\rvert$ | 0.0680 | 0.0679 | 0.0489 | 0.0459 | 0.0460 | 0.0461 | 0.0463 | 0.0460 | 0.0451 |

**Table 7**: Parameters of the in-sample estimations for the nine one-factor models, periods, and goodness of fit measures. The sample covers the period 2013-2018. In parenthesis we show the standard deviations of the estimations.

SCR, seasonality, and long-term oscillations. Sample period: 2013-2018.

| $\sum_t \min SCR(\hat{\theta}_t)$ | C2 | C3 | C4 | C5 | C6 |
|---|---|---|---|---|---|
| **Model 1** | 5.98 | 13.17 | 10.76 | 10.56 | 16.95 |
| **Model 2** | 5.99 | 13.19 | 10.72 | 10.50 | 16.87 |
| Seasonality (years) | 1 | 1 | 1 | 1 | 1 |
| Improvement over model 1 (%) | -0.21 | -0.16 | 0.32 | 0.53 | 0.47 |
| **Model 3** | 5.79 | 12.47 | 8.04 | 4.96 | 8.47 |
| Long-term swing (years) | 10 | 10 | 10 | 10 | 10 |
| Improvement over model 2 (%) | 3.35 | 5.45 | 25.05 | 52.75 | 49.79 |
| **Model 4** | 5.30 | 10.84 | 6.92 | 4.45 | 7.03 |
| Long-term swing (years) | 11.5 | 11.5 | 11.5 | 11.5 | 11.5 |
| Seasonality (years) | 7 | 7 | 7.5 | 7.5 | 8 |
| Improvement over model 3 (%) | 8.46 | 13.09 | 13.92 | 10.23 | 17.07 |
| **Model 5** | 5.29 | 10.70 | 6.91 | 4.28 | 7.00 |
| Long-term swing (years) | 13.5 | 13.5 | 13.5 | 13.5 | 13.5 |
| Seasonality (years) | 4 | 4 | 4.5 | 1.5 | 6 |
| Seasonality (years) | 4 | 4 | 4.5 | 6 | 6 |
| Improvement over model 4 (%) | 0.22 | 1.30 | 0.14 | 3.85 | 0.38 |
| **Model 6** | 5.37 | 10.65 | 6.90 | 4.32 | 6.92 |
| Long-term swing (years) | 13.5 | 13.5 | 13.5 | 13.5 | 13.5 |
| Seasonality (years) | 3.5 | 3 | 3.5 | 5 | 4 |
| Seasonality (years) | 3.5 | 3 | 3.5 | 2 | 4 |
| Seasonality (years) | 3.5 | 2.5 | 3 | 2 | 4.5 |
| Improvement over model 5 (%) | -1.51 | 0.47 | 0.14 | -0.93 | 1.14 |
| **Model 7** | 5.44 | 11.14 | 7.05 | 4.38 | 7.17 |
| Seasonality (years) | 4.5 | 4.5 | 5 | 5 | 5 |
| Improvement over model 2 (%) | 9.18 | 15.54 | 34.24 | 58.29 | 57.50 |
| **Model 8** | 5.29 | 10.67 | 6.91 | 4.45 | 6.98 |
| Seasonality (years) | 8 | 9 | 6.5 | 6 | 6 |
| Seasonality (years) | 8 | 9 | 7.5 | 6 | 6 |
| Improvement over model 7 (%) | 2.76 | 4.22 | 1.99 | -1.60 | 2.65 |
| **Model 9** | 5.27 | 10.55 | 6.77 | 4.35 | 6.79 |
| Seasonality (years) | 5 | 8.5 | 7 | 6.5 | 6.5 |
| Seasonality (years) | 5.5 | 9 | 7 | 6.5 | 6 |
| Seasonality (years) | 4.5 | 6 | 1 | 1 | 1 |
| Improvement over model 8 (%) | 0.38 | 1.12 | 2.03 | 2.25 | 2.72 |

**Table 8**: Sum of squared pricing errors, seasonal periods, long-term swing in the mean reversion level, and statistical improvement of each model. The sample period is 2013-2018.

Ranking of models, from lowest to highest SCR.

| Years | Whole sample | Until 2013 | 2013-2018 | 2013-2014 | 2014-2015 | 2015-2016 | 2016-2017 |
|---|---|---|---|---|---|---|---|
| | 9 (57.01) | 9 (14.90) | 9 (33.75) | 9 (13.60) | 9 (4.73) | 9 (0.94) | 9 (4.28) |
| | 6 (57.83) | 5 (15.15) | 6 (34.17) | 8 (13.71) | 6 (5.01) | 8(0.97) | 8 (4.41) |
| | 8 (58.33) | 6 (15.19) | 5 (34.18) | 6 (13.73) | 8 (5.09) | 5 (0.97) | 4 (4.69) |
| | 5 (58.43) | 8 (16.94) | 8 (34.29) | 5 (13.91) | 5 (5.09) | 6 (0.97) | 6 (4.75) |
| | 4 (59.76) | 4 (18.12) | 4 (34.54) | 4 (14.29) | 4 (5.14) | 4(0.99) | 5 (4.86) |
| | 7 (64.21) | 7 (19.53) | 7 (35.17) | 7 (15.75) | 7 (5.15) | 7 (1.02) | 7 (4.86) |
| | 3 (67.21) | 1 (24.26) | 3 (39.73) | 2 (18.07) | 3 (5.61) | 3 (1.50) | 3 (5.44) |
| | 2 (121.24) | 3 (21.81) | 2 (57.27) | 3 (18.34) | 2 (5.62) | 2 (1.39) | 2 (5.51) |
| | 1 (122.26) | 2 (21.88) | 1 (57.41) | 1 (18.77) | 1 (5.98) | 1 (1.63) | 1 (5.54) |

**Table 9**: Ranking of models, from lowest to highest squared sum of errors, for the periods indicated in the table.

Out-of-sample estimation for 2016 using data from 2015.

| **Model 1** | Q1 | Q2 | Q3 | Q4 | $\sum u^2$ | **Model 4** | Q1 | Q2 | Q3 | Q4 | $\sum u^2$ | **Model 7** | Q1 | Q2 | Q3 | Q4 | $\sum u^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| c2 | 0.3095 | 0.5047 | 0.1808 | 0.1637 | 1.1587 | c2 | 0.1918 | 0.6008 | 0.0640 | 0.1681 | 1.0247 | c2 | 0.1839 | 0.5080 | 0.1647 | 0.1729 | 1.0295 |
| c3 | 0.2628 | 0.2761 | 0.1503 | 0.2105 | 0.8997 | c3 | 0.2302 | 0.3643 | 0.0838 | 0.3209 | 0.9992 | c3 | 0.1946 | 0.2803 | 0.1494 | 0.2069 | 0.8312 |
| c4 | 0.1139 | 0.0419 | 0.0364 | 0.1088 | 0.3010 | c4 | 0.1478 | 0.1305 | 0.0554 | 0.1822 | 0.5159 | c4 | 0.1028 | 0.0464 | 0.0373 | 0.1125 | 0.299 |
| c5 | 0.1471 | 0.0270 | 0.0356 | 0.0870 | 0.2967 | c5 | 0.2178 | 0.0581 | 0.0629 | 0.1792 | 0.5180 | c5 | 0.1580 | 0.0356 | 0.0354 | 0.0895 | 0.3185 |
| c6 | 0.1022 | 0.0865 | 0.0750 | 0.1248 | 0.3885 | c6 | 0.1487 | 0.0222 | 0.0881 | 0.2023 | 0.4613 | c6 | 0.0999 | 0.0807 | 0.0711 | 0.1199 | 0.3716 |
| $\sum u^2$ | 0.9355 | 0.9362 | 0.4781 | 0.6948 | 3.0446 | $\sum u^2$ | 0.9363 | 1.1759 | 0.3542 | 1.0526 | 3.519 | $\sum u^2$ | 0.7392 | 0.9511 | 0.4579 | 0.7018 | 2.85 |

| **Model 2** | Q1 | Q2 | Q3 | Q4 | $\sum u^2$ | **Model 5** | Q1 | Q2 | Q3 | Q4 | $\sum u^2$ | **Model 8** | Q1 | Q2 | Q3 | Q4 | $\sum u^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| c2 | 0.4351 | 0.7105 | 0.1991 | 0.2249 | 1.5696 | c2 | 0.1803 | 0.4125 | 0.0815 | 0.1878 | 0.8621 | c2 | 0.3138 | 0.7404 | 0.2277 | 0.2080 | 1.4899 |
| c3 | 0.3443 | 0.4176 | 0.1731 | 0.2020 | 1.1370 | c3 | 0.2201 | 0.1190 | 0.1091 | 0.3981 | 0.8463 | c3 | 0.2699 | 0.4543 | 0.1377 | 0.1559 | 1.0178 |
| c4 | 0.1600 | 0.0867 | 0.0483 | 0.1101 | 0.4051 | c4 | 0.1449 | 0.0557 | 0.0733 | 0.1869 | 0.4608 | c4 | 0.1396 | 0.1128 | 0.0222 | 0.0986 | 0.3732 |
| c5 | 0.1640 | 0.0256 | 0.0327 | 0.0789 | 0.3012 | c5 | 0.2077 | 0.0412 | 0.0831 | 0.1829 | 0.5149 | c5 | 0.1796 | 0.0404 | 0.0562 | 0.1137 | 0.3899 |
| c6 | 0.0945 | 0.0907 | 0.1006 | 0.1857 | 0.4715 | c6 | 0.1443 | 0.0113 | 0.0984 | 0.2069 | 0.4609 | c6 | 0.1009 | 0.0247 | 0.0195 | 0.1157 | 0.2608 |
| $\sum u^2$ | 1.1980 | 1.3311 | 0.5538 | 0.8016 | 3.8845 | $\sum u^2$ | 0.8974 | 0.6397 | 0.4455 | 1.1626 | 3.1452 | $\sum u^2$ | 1.0038 | 1.3726 | 0.4632 | 0.6918 | 3.5314 |

| **Model 3** | Q1 | Q2 | Q3 | Q4 | $\sum u^2$ | **Model 6** | Q1 | Q2 | Q3 | Q4 | $\sum u^2$ | **Model 9** | Q1 | Q2 | Q3 | Q4 | $\sum u^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| c2 | 0.1775 | 0.5466 | 0.0959 | 0.2135 | 1.0355 | c2 | 0.2114 | 0.2513 | 0.2351 | 0.1617 | 0.8595 | c2 | 0.74298 | 0.7263 | 0.0564 | 0.0954 | 1.3079 |
| c3 | 0.2166 | 0.3964 | 0.1334 | 0.3707 | 1.1171 | c3 | 0.2583 | 0.1432 | 0.3787 | 0.3307 | 1.1109 | c3 | 0.3256 | 0.4842 | 0.0687 | 0.1986 | 1.0771 |
| c4 | 0.1309 | 0.1586 | 0.0810 | 0.2630 | 0.6341 | c4 | 0.1799 | 0.1366 | 0.0308 | 0.1689 | 0.5162 | c4 | 0.2249 | 0.1692 | 0.0371 | 0.0944 | 0.5256 |
| c5 | 0.1873 | 0.0789 | 0.0796 | 0.3116 | 0.6574 | c5 | 0.2603 | 0.0674 | 0.0409 | 0.1966 | 0.5652 | c5 | 0.2059 | 0.0417 | 0.0444 | 0.1355 | 0.4275 |
| c6 | 0.1292 | 0.0290 | 0.0802 | 0.3477 | 0.5861 | c6 | 0.1741 | 0.0297 | 0.0560 | 0.1797 | 0.4395 | c6 | 0.1050 | 0.0140 | 0.0387 | 0.1065 | 0.2642 |
| $\sum u^2$ | 0.8414 | 1.2095 | 0.4702 | 1.5085 | 4.0296 | $\sum u^2$ | 1.0840 | 0.6282 | 0.7415 | 1.0377 | 3.4914 | $\sum u^2$ | 1.2912 | 1.4355 | 0.2453 | 0.6304 | 3.6024 |

**Table 10**: Squared sum of errors of the out-of-sample estimations for each model for 2016 using data from 2015. Red numbers are the lowest squared errors.

Out-of-sample estimation for 2016 using data from 2014 and 2015.

| **Model 1** | Q1 | Q2 | Q3 | Q4 | $\sum u^2$ | **Model 4** | Q1 | Q2 | Q3 | Q4 | $\sum u^2$ | **Model 7** | Q1 | Q2 | Q3 | Q4 | $\sum u^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| c2 | 0.3369 | 1.1168 | 0.5984 | 0.3756 | 2.4277 | c2 | 0.1609 | 0.8292 | 0.2112 | 0.2532 | 1.4545 | c2 | 0.4077 | 1.0941 | 0.4959 | 0.3470 | 2.3447 |
| c3 | 0.2604 | 0.6553 | 0.4397 | 0.2749 | 1.6303 | c3 | 0.1769 | 0.5729 | 0.2286 | 0.3537 | 1.3321 | c3 | 0.2778 | 0.6529 | 0.4795 | 0.2643 | 1.6745 |
| c4 | 0.0729 | 0.3036 | 0.1923 | 0.1255 | 0.6943 | c4 | 0.0749 | 0.3539 | 0.1879 | 0.2519 | 0.8686 | c4 | 0.0647 | 0.3154 | 0.1940 | 0.1316 | 0.7057 |
| c5 | 0.0636 | 0.2347 | 0.1625 | 0.1003 | 0.5611 | c5 | 0.1169 | 0.3048 | 0.1921 | 0.2384 | 0.8522 | c5 | 0.08550 | 0.2285 | 0.1720 | 0.1027 | 0.5882 |
| c6 | 0.0638 | 0.2496 | 0.2379 | 0.2216 | 0.7729 | c6 | 0.0622 | 0.2483 | 0.1919 | 0.2452 | 0.7476 | c6 | 0.0485 | 0.2314 | 0.2913 | 0.2316 | 0.8028 |
| $\sum u^2$ | 0.7976 | 2.5600 | 1.6309 | 1.0979 | 6.0864 | $\sum u^2$ | 0.5919 | 2.3091 | 1.0111 | 1.3424 | 5.2545 | $\sum u^2$ | 0.8837 | 2.5222 | 1.6327 | 1.0772 | 6.1158 |
| **Model 2** | Q1 | Q2 | Q3 | Q4 | $\sum u^2$ | **Model 5** | Q1 | Q2 | Q3 | Q4 | $\sum u^2$ | **Model 8** | Q1 | Q2 | Q3 | Q4 | $\sum u^2$ |
| c2 | 0.5661 | 1.2537 | 0.5574 | 0.3953 | 2.7825 | c2 | 0.0936 | 0.7738 | 0.2254 | 0.2889 | 1.3817 | c2 | 0.3895 | 1.4258 | 0.6547 | 0.8427 | 3.3127 |
| c3 | 0.4180 | 0.7388 | 0.4257 | 0.2226 | 1.8551 | c3 | 0.1511 | 0.5703 | 0.2289 | 0.4654 | 1.4157 | c3 | 0.3561 | 0.8740 | 0.5166 | 0.4591 | 2.2058 |
| c4 | 0.1430 | 0.3274 | 0.1931 | 0.1305 | 0.7940 | c4 | 0.0711 | 0.4693 | 0.2021 | 0.2244 | 0.9660 | c4 | 0.0614 | 0.2984 | 0.1788 | 0.0679 | 0.6065 |
| c5 | 0.0850 | 0.2336 | 0.1593 | 0.1022 | 0.5801 | c5 | 0.1115 | 0.4314 | 0.2287 | 0.2095 | 0.9811 | c5 | 0.1222 | 0.2990 | 0.1397 | 0.1367 | 0.6976 |
| c6 | 0.0662 | 0.2547 | 0.2457 | 0.2900 | 0.8566 | c6 | 0.0670 | 0.3382 | 0.2055 | 0.2271 | 0.8378 | c6 | 0.0490 | 0.1649 | 0.2083 | 0.0681 | 0.4903 |
| $\sum u^2$ | 1.2884 | 2.8082 | 1.5812 | 1.1906 | 6.8684 | $\sum u^2$ | 0.4944 | 2.5830 | 1.0897 | 1.4154 | 5.5825 | $\sum u^2$ | 0.9781 | 3.0620 | 1.6980 | 1.5745 | 7.3126 |
| **Model 3** | Q1 | Q2 | Q3 | Q4 | $\sum u^2$ | **Model 6** | Q1 | Q2 | Q3 | Q4 | $\sum u^2$ | **Model 9** | Q1 | Q2 | Q3 | Q4 | $\sum u^2$ |
| c2 | 0.2007 | 0.7673 | 0.2147 | 0.2496 | 1.4323 | c2 | 0.2242 | 1.2140 | 0.4545 | 0.1872 | 2.0799 | c2 | 0.4089 | 1.4193 | 0.4629 | 0.3802 | 2.6713 |
| c3 | 0.1958 | 0.5393 | 0.2394 | 0.3600 | 1.3345 | c3 | 0.1774 | 0.7588 | 0.4973 | 0.2751 | 1.7086 | c3 | 0.2343 | 0.9294 | 0.3524 | 0.2240 | 1.7401 |
| c4 | 0.0777 | 0.3424 | 0.1887 | 0.2511 | 0.8599 | c4 | 0.0674 | 0.5031 | 0.1363 | 0.1984 | 0.9052 | c4 | 0.1817 | 0.3486 | 0.2109 | 0.1001 | 0.8413 |
| c5 | 0.1064 | 0.3052 | 0.1934 | 0.2395 | 0.8445 | c5 | 0.1147 | 0.3650 | 0.1638 | 0.1872 | 0.8307 | c5 | 0.1734 | 0.2254 | 0.1425 | 0.0894 | 0.6307 |
| c6 | 0.0570 | 0.2476 | 0.1903 | 0.2467 | 0.7416 | c6 | 0.0510 | 0.3181 | 0.1333 | 0.1985 | 0.7009 | c6 | 0.0492 | 0.1747 | 0.1733 | 0.1016 | 0.4988 |
| $\sum u^2$ | 0.6376 | 2.2017 | 1.0265 | 1.3469 | 5.2127 | $\sum u^2$ | 0.6348 | 3.1591 | 1.3852 | 1.0465 | 6.2256 | $\sum u^2$ | 1.0475 | 3.0974 | 1.3421 | 0.8953 | 6.3823 |

**Table 11**: Squared sum of errors of the out-of-sample estimations for each model for 2016 using data from 2014 and 2015. Red numbers are the lowest squared errors.

Out-of-sample estimation for 2017 using data from 2016.

| **Model 1** | Q1 | Q2 | Q3 | Q4 | $\sum u^2$ | **Model 4** | Q1 | Q2 | Q3 | Q4 | $\sum u^2$ | **Model 7** | Q1 | Q2 | Q3 | Q4 | $\sum u^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| c2 | 0.3018 | 0.0304 | 0.0640 | 0.2668 | 0.6630 | c2 | 0.3009 | 0.0304 | 0..0673 | 0.1063 | 0.5049 | c2 | 0.3012 | 0.0304 | 0.0594 | 0.2594 | 0.6506 |
| c3 | 0.2993 | 0.0302 | 0.0634 | 0.2663 | 0.6592 | c3 | 0.3004 | 0.0303 | 0.0648 | 0.1084 | 0.5039 | c3 | 0.2982 | 0.0304 | 0.0646 | 0.2645 | 0.6571 |
| c4 | 0.3006 | 0.0308 | 0.0637 | 0.2689 | 0.6640 | c4 | 0.2993 | 0.0306 | 0.0645 | 0.1126 | 0.5070 | c4 | 0.3044 | 0.0307 | 0.0668 | 0.2693 | 0.6712 |
| c5 | 0.3029 | 0.0309 | 0.0662 | 0.2687 | 0.6687 | c5 | 0.2987 | 0.0310 | 0.0633 | 0.1078 | 0.5008 | c5 | 0.3038 | 0.0310 | 0.0710 | 0.2684 | 0.6742 |
| c6 | 0.3041 | 0.0311 | 0.0673 | 0.2685 | 0.6710 | c6 | 0.3199 | 0.0312 | 0.0648 | 0.1116 | 0.5275 | c6 | 0.3060 | 0.0310 | 0.0702 | 0.2655 | 0.6727 |
| $\sum u^2$ | 1.5087 | 0.1535 | 0.3246 | 1.3392 | 3.3260 | $\sum u^2$ | 1.5193 | 0.1535 | 0.3246 | 0.5468 | 2.5442 | $\sum u^2$ | 1.5137 | 0.1536 | 0.3315 | 1.3273 | 3.3261 |
| **Model 2** | Q1 | Q2 | Q3 | Q4 | $\sum u^2$ | **Model 5** | Q1 | Q2 | Q3 | Q4 | $\sum u^2$ | **Model 8** | Q1 | Q2 | Q3 | Q4 | $\sum u^2$ |
| c2 | 0.3019 | 0.0302 | 0.0638 | 0.2682 | 0.6641 | c2 | 0.3032 | 0.0314 | 0.0642 | 0.2387 | 0.6375 | c2 | 0.3018 | 0.0303 | 0.0573 | 0.2661 | 0.6555 |
| c3 | 0.2994 | 0.0301 | 0.0634 | 0.2670 | 0.6599 | c3 | 0.2935 | 0.0334 | 0.0652 | 0.3044 | 0.6965 | c3 | 0.2966 | 0.0374 | 0.0592 | 0.2659 | 0.6591 |
| c4 | 0.3005 | 0.0308 | 0.0638 | 0.2688 | 0.6639 | c4 | 0.2997 | 0.0324 | 0.0614 | 0.2488 | 0.6423 | c4 | 0.3032 | 0.0306 | 0.0612 | 0.2729 | 0.6679 |
| c5 | 0.3028 | 0.0311 | 0.0664 | 0.2682 | 0.6685 | c5 | 0.3005 | 0.0324 | 0.0667 | 0.2485 | 0.6481 | c5 | 0.3122 | 0.0309 | 0.0908 | 0.2800 | 0.7139 |
| c6 | 0.3040 | 0.0313 | 0.0675 | 0.2680 | 0.6708 | c6 | 0.3226 | 0.0335 | 0.0787 | 0.2619 | 0.6867 | c6 | 0.3115 | 0.0314 | 0.1148 | 0.2712 | 0.7294 |
| $\sum u^2$ | 1.5086 | 0.1535 | 0.3249 | 1.3402 | 3.3272 | $\sum u^2$ | 1.5195 | 0.1631 | 0.3362 | 1.2924 | 3.3112 | $\sum u^2$ | 1.5253 | 0.1611 | 0.3833 | 1.3561 | 3.4258 |
| **Model 3** | Q1 | Q2 | Q3 | Q4 | $\sum u^2$ | **Model 6** | Q1 | Q2 | Q3 | Q4 | $\sum u^2$ | **Model 9** | Q1 | Q2 | Q3 | Q4 | $\sum u^2$ |
| c2 | 0.2958 | 0.0304 | 0.0577 | 0.2679 | 0.6518 | c2 | 0.3064 | 0.0321 | 0.0639 | 0.2653 | 0.6677 | c2 | 0.3031 | 0.0312 | 0.0630 | 0.1934 | 0.5907 |
| c3 | 0.2950 | 0.0302 | 0.0638 | 0.2673 | 0.6563 | c3 | 0.3217 | 0.0340 | 0.0564 | 0.2664 | 0.6785 | c3 | 0.2995 | 0.0377 | 0.0657 | 0.1918 | 0.5947 |
| c4 | 0.2979 | 0.0308 | 0.0718 | 0.2696 | 0.6701 | c4 | 0.3050 | 0.0320 | 0.0575 | 0.2719 | 0.6664 | c4 | 0.3004 | 0.0307 | 0.0635 | 0.1977 | 0.5923 |
| c5 | 0.3021 | 0.0309 | 0.0882 | 0.2691 | 0.6903 | c5 | 0.3078 | 0.0317 | 0.0906 | 0.2732 | 0.7033 | c5 | 0.2966 | 0.0351 | 0.0725 | 0.1987 | 0.6029 |
| c6 | 0.3048 | 0.0311 | 0.1048 | 0.2685 | 0.7092 | c6 | 0.3085 | 0.0366 | 0.0870 | 0.2737 | 0.7058 | c6 | 0.3144 | 0.0469 | 0.0639 | 0.1993 | 0.6245 |
| $\sum u^2$ | 1.4956 | 0.1534 | 0.3862 | 1.3423 | 3.3775 | $\sum u^2$ | 1.5495 | 0.1663 | 0.3555 | 1.3505 | 3.4218 | $\sum u^2$ | 1.5139 | 0.1816 | 0.3286 | 0.9810 | 3.0051 |

**Table 12**: Squared sum of errors of the out-of-sample estimation for each model for 2017 using data from 2016. Red numbers are the lowest squared errors.

Out-of-sample estimation for 2017 using data from 2015 and 2016.

| **Model 1** | Q1 | Q2 | Q3 | Q4 | $\sum u^2$ | **Model 4** | Q1 | Q2 | Q3 | Q4 | $\sum u^2$ | **Model 7** | Q1 | Q2 | Q3 | Q4 | $\sum u^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C2 | 0.1996 | 0.1612 | 0.1791 | 0.0605 | 0.6004 | c2 | 0.1700 | 0.1502 | 0.1832 | 0.0610 | 0.5644 | c2 | 0.2042 | 0.1589 | 0.1818 | 0.0609 | 0.6058 |
| c3 | 0.2025 | 0.1622 | 0.1799 | 0.0613 | 0.6059 | c3 | 0.1605 | 0.1476 | 0.1838 | 0.0612 | 0.5531 | c3 | 0.2034 | 0.1645 | 0.1820 | 0.0618 | 0.6117 |
| c4 | 0.2085 | 0.1638 | 0.1796 | 0.0610 | 0.6129 | c4 | 0.1599 | 0.1464 | 0.1842 | 0.0605 | 0.551 | c4 | 0.2070 | 0.1612 | 0.1807 | 0.0611 | 0.6100 |
| c5 | 0.2172 | 0.1660 | 0.1797 | 0.0610 | 0.6239 | c5 | 0.1591 | 0.1469 | 0.1846 | 0.0607 | 0.5513 | c5 | 0.2140 | 0.1622 | 0.1795 | 0.0628 | 0.6185 |
| c6 | 0.2232 | 0.1694 | 0.1773 | 0.0616 | 0.6315 | c6 | 0.1612 | 0.1481 | 0.1830 | 0.0621 | 0.5544 | c6 | 0.2288 | 0.1642 | 0.1761 | 0.0608 | 0.6299 |
| $\sum u^2$ | 1.0510 | 0.8226 | 0.8956 | 0.3054 | 3.074 | $\sum u^2$ | 0.8106 | 0.7393 | 0.9188 | 0.3055 | 2.7742 | $\sum u^2$ | 1.0574 | 0.8111 | 0.9001 | 0.3074 | 3.076 |

| **Model 2** | Q1 | Q2 | Q3 | Q4 | $\sum u^2$ | **Model 5** | Q1 | Q2 | Q3 | Q4 | $\sum u^2$ | **Model 8** | Q1 | Q2 | Q3 | Q4 | $\sum u^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| c2 | 0.2157 | 0.1574 | 0.1805 | 0.0607 | 0.6148 | c2 | 0.1623 | 0.1465 | 0.1797 | 0.0571 | 0.5456 | c2 | 0.2165 | 0.1629 | 0.1817 | 0.0609 | 0.6220 |
| c3 | 0.2046 | 0.1596 | 0.1807 | 0.0608 | 0.6057 | c3 | 0.1500 | 0.1511 | 0.1803 | 0.0576 | 0.5390 | c3 | 0.2054 | 0.1649 | 0.1818 | 0.0609 | 0.6130 |
| c4 | 0.2047 | 0.1634 | 0.1797 | 0.0605 | 0.6083 | c4 | 0.1491 | 0.1443 | 0.1793 | 0.0573 | 0.5300 | c4 | 0.2165 | 0.1680 | 0.1811 | 0.0604 | 0.6260 |
| c5 | 0.2270 | 0.1682 | 0.1787 | 0.0608 | 0.6347 | c5 | 0.1601 | 0.1435 | 0.1778 | 0.0608 | 0.5422 | c5 | 0.2158 | 0.1677 | 0.1807 | 0.0616 | 0.6258 |
| c6 | 0.2543 | 0.1740 | 0.1764 | 0.0611 | 0.6658 | c6 | 0.1831 | 0.1501 | 0.1772 | 0.0604 | 0.5708 | c6 | 0.2328 | 0.1728 | 0.1779 | 0.0625 | 0.6460 |
| $\sum u^2$ | 1.1063 | 0.8231 | 0.8960 | 0.3038 | 3.1292 | $\sum u^2$ | 0.8046 | 0.7355 | 0.8942 | 0.2932 | 2.7275 | $\sum u^2$ | 1.0869 | 0.8363 | 0.9031 | 0.3063 | 3.1326 |

| **Model 3** | Q1 | Q2 | Q3 | Q4 | $\sum u^2$ | **Model 6** | Q1 | Q2 | Q3 | Q4 | $\sum u^2$ | **Model 9** | Q1 | Q2 | Q3 | Q4 | $\sum u^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| c2 | 0.1583 | 0.1435 | 0.1778 | 0.0613 | 0.5409 | c2 | 0.1942 | 0.1359 | 0.1771 | 0.0607 | 0.5679 | c2 | 0.2001 | 0.1636 | 0.1802 | 0.0622 | 0.6061 |
| c3 | 0.1492 | 0.1425 | 0.1776 | 0.0609 | 0.5302 | c3 | 0.1924 | 0.1441 | 0.1799 | 0.0607 | 0.5771 | c3 | 0.2006 | 0.1640 | 0.1816 | 0.0603 | 0.6065 |
| c4 | 0.1462 | 0.1435 | 0.1766 | 0.0604 | 0.5267 | c4 | 0.1917 | 0.1368 | 0.1757 | 0.0599 | 0.5641 | c4 | 0.2100 | 0.1660 | 0.1809 | 0.0598 | 0.6167 |
| c5 | 0.1637 | 0.1457 | 0.1755 | 0.0605 | 0.5454 | c5 | 0.1952 | 0.1442 | 0.1748 | 0.0614 | 0.5756 | c5 | 0.2079 | 0.1754 | 0.1802 | 0.0612 | 0.6247 |
| c6 | 0.2002 | 0.1505 | 0.1728 | 0.0608 | 0.5843 | c6 | 0.2802 | 0.1431 | 0.1733 | 0.0628 | 0.6594 | c6 | 0.1994 | 0.1750 | 0.1776 | 0.0607 | 0.6127 |
| $\sum u^2$ | 0.8178 | 0.7256 | 0.8803 | 0.3039 | 2.7276 | $\sum u^2$ | 1.0537 | 0.7041 | 0.8807 | 0.3054 | 2.9439 | $\sum u^2$ | 1.0180 | 0.8439 | 0.9005 | 0.3042 | 3.0666 |

**Table 13**: Squared sum of errors of the out-of-sample estimation for each model for 2017 using data from 2015 and 2016. Red numbers are the lowest squared errors.

## Appendix of Figures

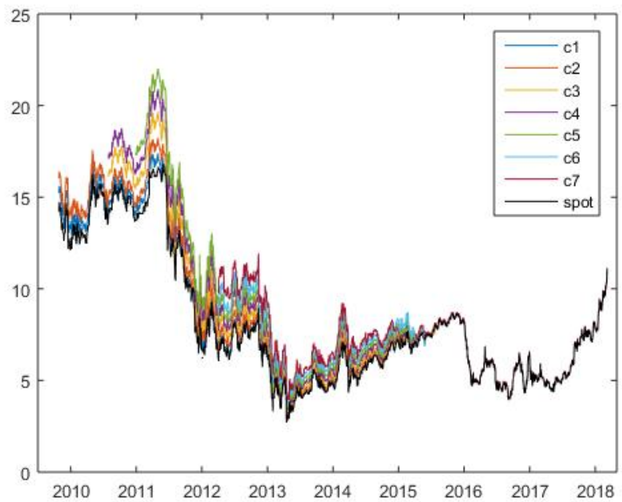


**Figure 1**: Time series of prices of futures c1 to c7 and spot price for the whole sample (first available data until 2018).

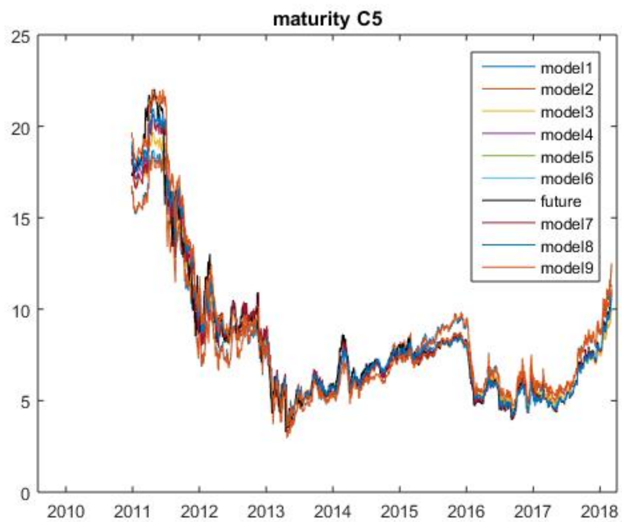


**Figure 2**: Spot price and fitted prices for the futures c5 under the nine models. The sample period for this futures is from 29, December, 2010 until 9, March, 2018.

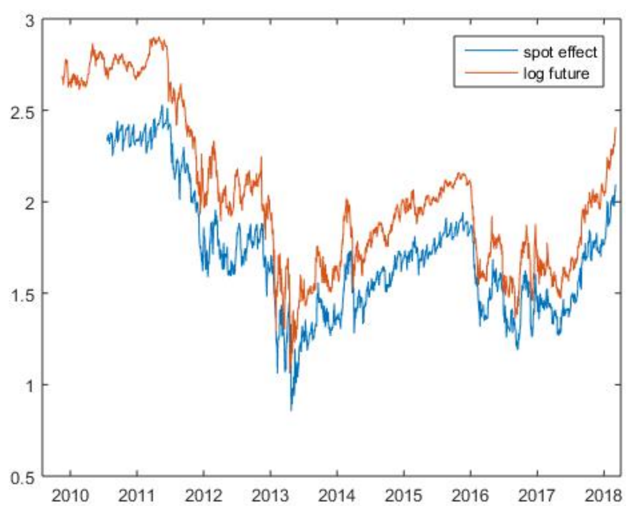


**Figure 3**: Spot effect and observed log-price of the futures c2. The sample period is from 27, October, 2009 until 9, March, 2018.

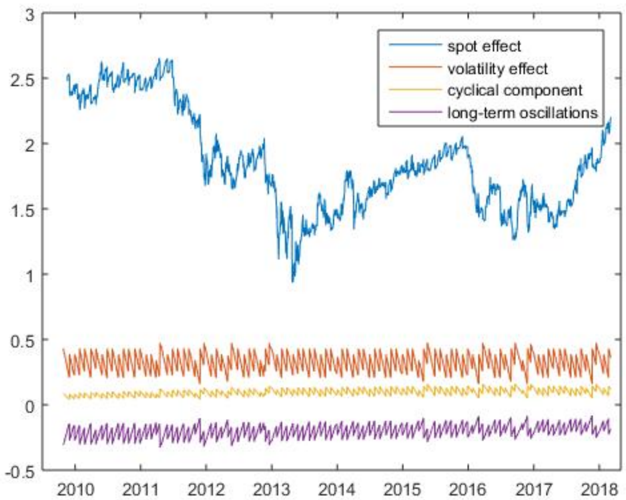


**Figure 4**: Decomposition of the four effects in model 5 for the futures c2. The sample period is from 27, October, 2009 until 9, March, 2018.

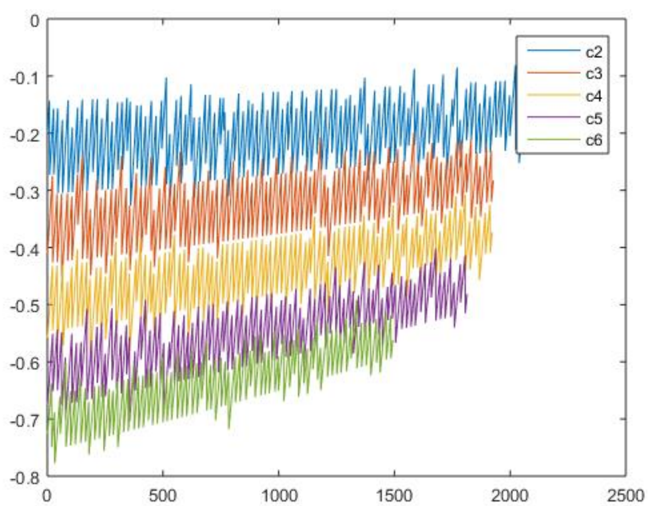


**Figure 5**: Estimation of the level of mean-reversion (long-term swing) for the prices of the futures c2 to c6.

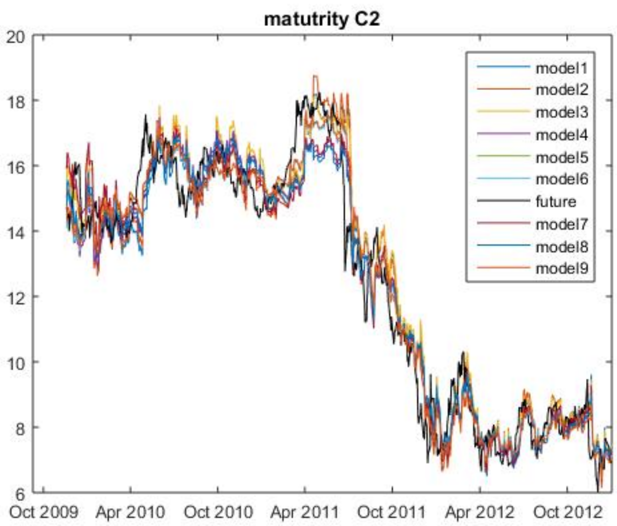


**Figure 6**: Spot price and fitted prices for the futures c2 under the nine models. The sample period for this futures is from 27, October, 2009 until 9, March, 2018.

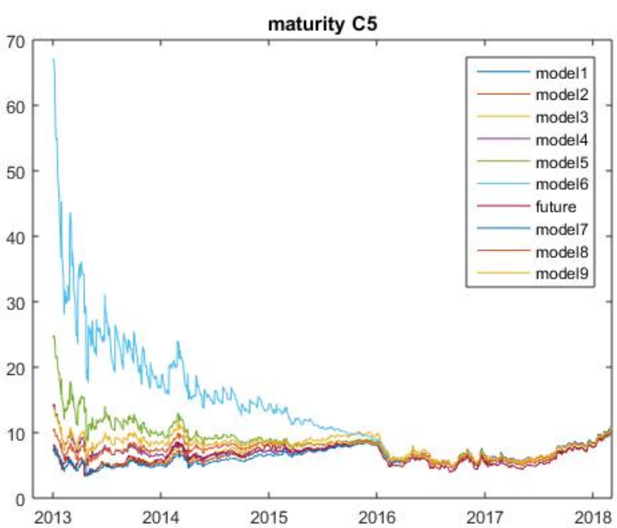


**Figure 7**: Spot price and fitted prices for the futures c5 under the nine models. The sample period for this futures is from 2013 until 9, March, 2018.