Net-Zero Transition Paths: Facts and Fiction

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Abstract

We develop a probabilistic model to analyze net-zero commitments of companies. It allows us to incorporate the inherent variability of emission reductions of a particular company. In our model, carbon emissions follow a geometric Brownian Motion, where the drift and the volatility are based on historical emission reduction rates. Using carbon emissions, the carbon budget can be defined, which is the cumulative amount of carbon emissions over a specified time range and can be used to dynamically track the remaining carbon budget prescribed by a net-zero scenario. The net-zero scenario pathway is calibrated to a firm of interest considering its emissions reduction targets, including the net-zero target. Using estimated carbon emissions' parameters, we compute the probability of the firm reaching its emission reduction targets. Moreover, we provide the probability of respecting the carbon budget implied by the calibrated net-zero pathway. Both probabilities are adjusted with the arrival of new emissions data. The time series of probabilities of respecting the pledged carbon budget can be used for monitoring and regulation of the corporate net-zero transition as well as to measure firm's exposure to climate risk.

Keywords: climate risk, resilience, net-zero path, carbon budget, sectoral scenarios

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1. Introduction and Motivation

According to the IPCC's glossary, IPCC (2018), net-zero carbon emissions are reached as soon as carbon emissions can be balanced out by carbon removals, which refer to the process of removing carbon emissions from the atmosphere such as, for instance, Carbon Capture and Storage, CCS. It is important to note that CCS, among other removal methods used in climate scenarios, is not yet available at scale and is rather expected to be used in greater, but limited, amounts in the future. Therefore, substantial carbon emission reductions are still to be made in order to be able to achieve net-zero emissions by 2050, pursuing the common goal of limiting the global average temperature rise to 1.5 °C as stated by the Paris Agreement and Glasgow Climate Pact among others, IPCC (2021). The IPCC's SSP1-1.9 scenario, for instance, requires an absolute reduction of carbon emissions of around 42% by 2030 and 95% by 2050, both compared to the carbon emissions' amount in 2020, IPCC (2021). Net-zero scenarios by other providers show similar scale of reductions. Thus, in this paper, we focus on absolute reductions of carbon emissions and do not consider any form of carbon removals.

Despite large emission reduction requirements in scenarios achieving net-zero emissions by 2050, pledges to arrive at this ambitious goal have gained momentum among companies. Specifically, net-zero targets have become increasingly popular. According to Hans et al. (2022), around one third of listed companies from Forbes Global 2000, a yearly list of 2000 largest companies, has a net-zero target as of 2022. The Science Based Targets initiative, SBTi, the initiative that allows companies to set targets, which it assesses to be aligned with climate science, has to date received around 5500 applications from companies willing to set science-based targets. The number of applicants doubles each year starting from the foundation of the initiative in 2015. Around 40% of all validated and committed targets are net-zero targets set by companies of different size including financial institutions.

It is debatable what drivers push companies to set such ambitious climate targets. Bolton and Kacperczyk (2022) argue, for instance, that the reasons might be an increasing awareness of the consequences of climate change and, hence, a greater incentive for action; a craving to capture investor's attention by becoming more sustainable; a pioneering climate performance and the need to advertise it, or just an attempt of green washing. Unfortunately, firms provide little to no articulation on their net-zero transition strategy that ensures the achievement of net-zero targets. Therefore, there is clearly a need to verify the feasibility of net-zero claims. In this work, we construct a calibrated net-zero pathway considering net-zero targets for a company of interest that approximates its pledged net-zero emission pathway. Using observed emission reduction rates, we can assess the probability of reaching net-zero targets and, thus, check their credibility in terms of historical performance. Moreover, we provide the probability of respecting the carbon budget implied by the calibrated net-zero pathway.

To approximate the pledged net-zero emission path for a firm, we complement its emission targets by a corresponding sectoral emission scenario calibrated to the firm level. We use sectoral scenarios to control for different decarbonization capacities among sectors. The scenarios are based on various socioeconomic assumptions and provide aggregated future emissions in a sector. For simplicity, we assume that activities of firms under consideration are homogeneous and a distinct sector can be clearly identified. The Network for Greening the Financial System, NGFS, provides sectoral scenarios within different integrated assessment models, IAMs. The sectoral scenarios from the NGFS yield the amount of absolute greenhouse gas emissions in a specific sector in periods of five years, assuming constant linear emission reductions in between these. The annual decarbonization rates of a scenario correspond to average annual decarbonization rates in a sector. As being predominantly above or below the sector average is costly, either because of the delayed or divergent transition, see, for instance, NGFS (2022), we assume that firms follow the sector average in between the targets. We, therefore, apply the same average decarbonization rates to the starting emissions value of a firm in order to calibrate the sectoral scenarios to the firm level. We further adjust the calibrated net-zero pathway to respect firm's emission reduction targets by varying the length of the linear periods.

We model carbon emissions as a geometric Brownian Motion. The drift and the volatility are based on historical emission reduction rates. Concerning carbon emissions, it is important to note that an analysis of these allows solely a static comparison with a target emission pathway and does not provide a comprehensive assessment of the observed net-zero transition. Nevertheless, carbon emissions serve as a foundation to construct the carbon budget, which is the integral of carbon emissions over a specified transition interval. The carbon budget can be used to dynamically track the remaining amount of carbon emissions prescribed by a net-zero scenario and consider both observed and future projected decarbonization efforts. It, thus, can be seen as a forward-looking and decision useful net-zero transition assessment

metric.
Our model can be used for monitoring and regulation of the corporate net-zero transition. Kiesel and Stahl (2022) argue that climate risks require a resilience-based approach that ensures the ability to recover after the damage induced by climate change. The authors advocate the use of the precommitment approach, PCA, to handle with unknown unknowns, which are dominant for climate risks. The committed carbon budget is the precommitment that companies make and that can be updated regularly. Guiding companies to improve the probability of achieving the most resilient climate scenarios and adjusting the emission pathway, if necessary, extend the PCA to a controllable dynamic model. Such a dynamic framework for supervising corporate resilience can be used by regulators and policymakers to limit climate risk. Furthermore, asset managers, who are increasingly interested in constructing long-term net-zero aligned portfolios, will profit from the flexibility of selecting companies which are net-zero aligned with a certain probability. Computing the overall expected carbon budget with a certain probability and comparing it with the calibrated net-zero budget allow for future decarbonization incentives and removals management of a portfolio of interest. Finally, the framework can be also used for controlling and adjusting of Scope 3 emissions, which are indirect emissions produced mostly by the value chain and the disclosure of which gradually becomes obligatory.
Our paper contributes to the literature on the analysis of net-zero transition and seeks to provide forward-looking and decision useful net-zero metric to assess the decarbonization performance with respect to the net-zero scenario. Le Guenedal et al. (2022) and Slimane et al. (2022), for instance, provide various net-zero carbon metrics applied to construct a net-zero aligned portfolio. The authors focus mostly on the emission reduction rate and its change during the transition. The introduced carbon budget approach allows to determine the relative position with respect to the net-zero scenario, where the future projections are made assuming a deterministic constant emissions trend. Considering the mentioned literature, our contribution lies especially in the stochastic modelling approach of carbon emissions and carbon budget. It allows us to incorporate the inherent variability of emission reductions. We, furthermore, are able to provide probabilities of staying below or at net-zero emissions as well as of respecting the carbon budget implied by the calibrated net-zero emission pathway. Representations of carbon emissions and carbon budget in our model are consistent with the deterministic version provided by Le Guenedal et al. (2022). Moreover, to our knowledge, we are first to

provide a calibration method that is based on sectoral emission scenarios and incorporates companies' self-set emission targets. In this way, we can better approximate the emission pathway that leads firms to the pledged emission reduction targets.
The remainder of this paper is structured as follows. In Chapter 2, we describe our modelling approach. Carbon emissions and the carbon budget are defined. Moreover, the distribution of the carbon budget is approximated using the log-normal distribution by moment matching. The accuracy of the approximation is verified using Monte Carlo simulation. The calibration of sectoral climate scenarios to the firm level follows in Chapter 3. Chapter 4 presents an application of the model. Especially, a time series of probabilities of reaching emission reduction targets as well as respecting the carbon budget implied by the calibrated net-zero emission pathway are computed for two simulated pathes as well as for a subset of firms from different sectors. Finally, Chapter 6 concludes.

2. Probabilistic Model

We assume that carbon emissions of firms follow a geometric Brownian Motion (gBM), where the drift and the volatility are based on historical emission reduction rates. It allows us to incorporate the inherent variability of emission reductions of a particular company. Using carbon emissions, the carbon budget can be defined, which is the integral of carbon emissions over a specified transition interval and be used to dynamically track the remaining carbon budget prescribed by a net-zero scenario.

2.1. Carbon Emission

For a firm that has started its transition to net-zero, let us assume that carbon emissions dynamics satisfy the following stochastic differential equation over the net-zero transition period $[t_0, T]$

$$\frac{d\mathcal{CE}(t)}{\mathcal{CE}(t)} = \mu\, dt + \sigma\, dW(t), \tag{1}$$

where $\mu < 0$ denotes the drift, $\sigma > 0$ is the volatility of historical emission reduction rates and $W(t)$ is a standard Brownian motion.
The solution to (1) can be easily obtained using Itô's lemma, so that carbon emissions read

$$\mathcal{CE}(t) = \mathcal{CE}(t_0) \exp\left[\left(\mu - \frac{\sigma^2}{2}\right)(t - t_0) + \sigma\left(W(t) - W(t_0)\right)\right], \tag{2}$$

where $\mathcal{CE}(t_0) > 0$ is the carbon emissions amount at time $t = t_0$.
We define the drift as

$$\mu := \log(1 - R), \tag{3}$$

where $0 < R < 1$ is a mean historical emission reduction rate. Using the definition of μ in (3) and assuming that $\sigma = 0$ in Equation (2), we arrive at a deterministic representation of emissions given by Le Guenedal et al. (2022):

$$\mathcal{CE}(t) = \mathcal{CE}(t_0)(1 - R)^{(t-t_0)}.$$

Note that $\mathcal{CE}(t)$ given by (2) and assuming (3) is log-normally distributed with the mean of

$$m(t_0, t) := \log \mathcal{CE}(t_0) + \left(\log(1 - R) - \frac{\sigma^2}{2}\right)(t - t_0)$$

and the standard deviation of

$$v(t_0, t) := \sigma\sqrt{t - t_0}.$$

The moments of $\mathcal{CE}(t)$ are given by

$$\mathbb{E}[\mathcal{CE}^s(t)] = \exp\left(sm(t_0, t) + \frac{s^2\, v^2(t_0, t)}{2}\right). \tag{4}$$

The first moment is, thus, equal to

$$\mathbb{E}[\mathcal{CE}(t)] = \mathcal{CE}(0)\,(1 - R)^{(t-t_0)}, \tag{5}$$

where we used the definition of $m(t_0, t)$ and $v(t_0, t)$. Proceeding similarly, the second moment reads

$$\mathbb{E}[\mathcal{CE}^2(t)] = \mathcal{CE}(t_0)\, \exp\Big(2\log(1 - R)(t - t_0) + \sigma^2(t - t_0)\Big). \tag{6}$$

Using (10), we can compute further moments.

2.2. Carbon Budget

We follow Le Guenedal et al. (2022) and define the carbon budget as the integral of carbon emissions. Using (2) and (3), the carbon budget over $[t_0, T]$ is

$$\mathcal{CB}(t_0, T) = \mathcal{CE}(t_0)\int_{t_0}^{T} \exp\left[\left(\log(1 - R) - \frac{\sigma^2}{2}\right)t + \sigma\, W_t\right] dt.$$

We next present the first two moments of the carbon budget. The first moment reads

$$\mathbb{E}[\mathcal{CB}(t_0,T)] = \frac{\mathcal{CE}(t_0)}{\log(1-R)} \left((1-R)^{(T-t_0)} - 1\right), \tag{7}$$

where we used the Fubini-Tonelli's theorem, the Equation (5) and simple integration. Again, the value of the carbon budget matches the deterministic version of the carbon budget given by Le Guenedal et al. (2022).
The second moment is more complicated and is given by

$$\begin{aligned}\mathbb{E}[\mathcal{CB}^2(t_0,T)] = \mathcal{CE}^2(t_0)\Bigg(& \frac{\exp\left((2\log(1-R)+\sigma^2)(T-t_0)\right)}{(\log(1-R)+\sigma^2/2)(\log(1-R)+\sigma^2)} \\ & - \frac{2\exp\left(\log(1-R)(T-t_0)\right)}{\log(1-R)(\log(1-R)+\sigma^2)} + \frac{1}{\log(1-R)(\log(1-R)+\sigma^2/2)}\Bigg),\end{aligned} \tag{8}$$

where it can be either computed using Fubini-Tonelli's theorem, the first two moments of carbon emissions and integration or by using a formula for moments of the sum of log-normals provided by Geman and Yor (1993).

2.3. Carbon Budget's Approximation

The distribution of the carbon budget is, unfortunately, not available in closed form. As a solution, literature suggests Monte Carlo estimation, numerical methods or analytical approximations among other techniques. Especially, an approximation with a log-normal distribution is a popular practice used for pricing Asian options, which pay off the average underlying price, see, for instance, Levy (1992), Schwartz and Yeh (1982), Fenton (1960), Turnbull and Wakeman (1991) among others. The Wilkinson- or Levy-approach, for instance, lies in matching the first two moments of the underlying sum of log-normals and, thus, determining the parameters for the log-normal distribution, see Levy (1992), Fenton (1960), Schwartz and Yeh (1982) among others. As this method provides good accuracy of the estimates for specific parameter ranges, we apply it to approximate the distribution of the carbon budget, which we use to compute the probability of staying below the carbon budget implied by the calibrated net-zero scenario. Comparing to Monte Carlo estimates the method is accurate up to an absolute difference of 3% in probabilities for different parameter combinations.

To perform the approximation, we assume that the carbon budget is log-normally distributed

$$\log CB(t_0, T) \sim \mathcal{N}(\bar{\mu}(t_0, T), \bar{v}^2(t_0, T)), \tag{9}$$

where the parameters can be found by moment matching. Assuming (9), the moments of carbon budget are given by

$$\mathbb{E}[\mathcal{CB}^m(t_0, T)] = \exp(m\bar{\mu}(t_0, T) + \frac{m^2\, \bar{v}^2(t_0, T)}{2}). \tag{10}$$

Consequently, we solve

$$\begin{cases} \mathbb{E}[\mathcal{CB}(t_0, T)] = \exp\left(\bar{\mu}(t_0, T) + \frac{\bar{v}^2(t_0,T)}{2}\right), \\ \mathbb{E}[\mathcal{CB}^2(t_0, T)] = \exp\left(2\bar{\mu}(t_0, T) + 2\,\bar{v}^2(t_0, T)\right), \end{cases}$$

by using (7) and (8) and obtain

$$\begin{aligned} \bar{\mu}(t_0, T) &= 2\log(\mathbb{E}[\mathcal{CB}(t_0, T)]) - \frac{1}{2}\log(\mathbb{E}[\mathcal{CB}^2(t_0, T)]), \\ \bar{v}^2(t_0, T) &= \log(\mathbb{E}[\mathcal{CB}^2(t_0, T)]) - 2\log(\mathbb{E}[\mathcal{CB}(t_0, T)]), \end{aligned}$$

where $\bar{\mu}(t_0, T)$ and $\bar{v}^2(t_0, T)$ are well-defined, see Appendix. We next compare the accuracy of such approximation procedure for our application using Monte Carlo estimates.

2.4. Monte Carlo Simulation

We assess the approximation error of the log-normal distribution for 20, 50 and 108 observations of carbon emissions and different parameter combinations, where $R = 0.01, 0.03, 0.07$ and $\sigma = 0.05, 0.1, 0.2, 0.3$. We simulate paths of the gBM by using the following discretized representation

$$\mathcal{CE}(t+1) = \mathcal{CE}(t)\exp\left[\left(\log(1-R) - \frac{\sigma^2}{2}\right)\Delta t + \sigma\sqrt{\Delta t}\,\epsilon_t\right],$$

where $\epsilon_t \sim \mathcal{N}(0, 1)$ are independently identically distributed for $0 \leq t < T$ and Δt is a step size of the discretization. We produce 200,000 iterations to minimize the standard error of Monte Carlo estimates.

To compute the carbon budget over each simulation, we integrate over it using the trapezoidal rule. Next, we assess the probability of staying below the carbon budget of the net-zero pathway

$$\mathbb{P}(\mathcal{CB}(0,T) \leq \tilde{b}^{NZ}(0,T)),$$

where $CB(0,T)$ is the carbon budget computed in a Monte Carlo simulation and $\tilde{b}^{NZ}(0,T))$ is the carbon budget of the net-zero pathway, which is a calibrated net-zero scenario pathway described in detail in Chapter 3. Tables B.1, B.2 and B.3 present the results of the approximation, Monte Carlo estimates with a corresponding standard error and an absolute difference between the approximation's value and the Monte Carlo estimate for different parameter combinations. Considering the empirically assessed parameters in Subsection 4.1, the absolute error of the log-normal approximation is less than 3%. We observe that the absolute deviation from the Monte Carlo estimate is increasing in σ and N, which is similar to the results given by Levy (1992). Moreover, in most of the cases, the probability of respecting the carbon budget is undervalued, except for big probability values, where it is overvalued.

3. Calibration of Emission Scenarios

Emission scenarios provide a set of emission projections arising from specific assumptions on emission drivers. Such scenarios are usually available globally, see among others IPCC's or IEA's scenarios. A special type of emission scenarios is, admittedly, represented by sectoral emission scenarios that consider emission reduction abilities of different sectors, see Figure 1. We use sectoral emission scenarios provided by the NGFS to construct calibrated emission scenario pathways for a company of interest. In addition, individual emission reduction targets can be incorporated into the calibrated net-zero emission scenario pathway.

3.1. Calibration to the Firm Level

The NGFS, provides sectoral emission scenarios within different integrated assessment models, IAMs. We use GCAM 5.3+ IAM, which provides the best sectoral granularity including Cement, Real Estate, Steel, Transportation, Agriculture and Forestry, Chemicals, Industry, Electricity and Other Energy Supply. Figure 2, for instance, presents four emission scenarios for the sector of Chemicals reaching net-zero emissions around 2045

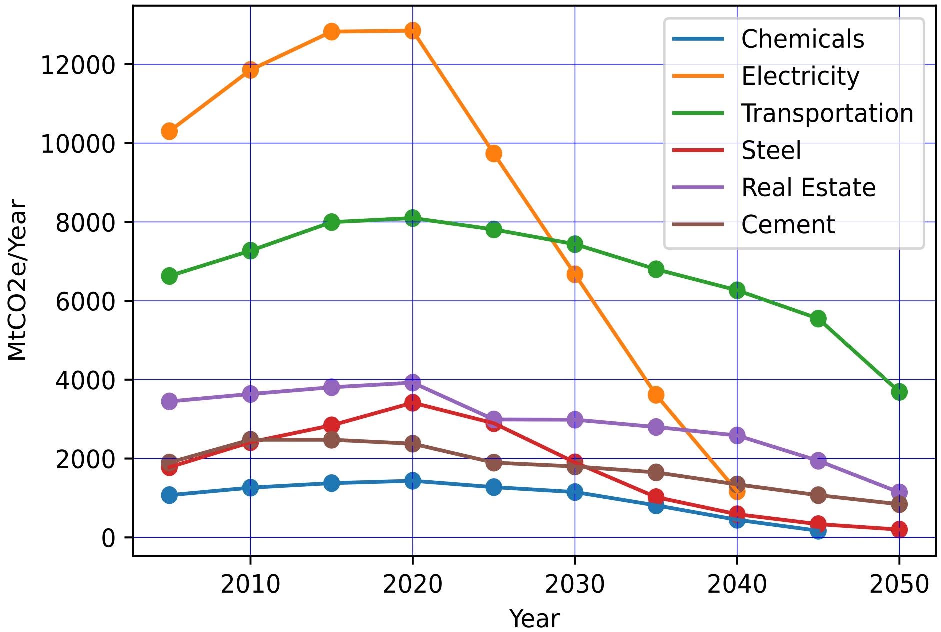


Figure 1: Net-Zero by 2050 Scenario for Different Sectors in GCAM 5.3+ Model provided by NGFS. Source: IIASA Scenario Explorer.

within the GCAM 5.3+ IAM, see NGFS (2022). Sectoral emission scenarios by the NGFS yield the amount of absolute greenhouse gas emissions in a specific sector in periods of five years, assuming constant absolute emission reductions inbetween these, see Figure 3. Annual decarbonization rates of a scenario correspond to average annual decarbonization rates in a sector. As being predominantly above or below the sector average is costly, either because of delayed or divergent transition, see for instance NGFS (2022), we assume firms in the underlying sector to follow the average sectoral emission pathway by mimicking its shape.

We fix a sector from a set of sectors under consideration denoted by S. In each scenario, $j \in J$, where J is the set of scenarios, we have a finite number of consecutive linear periods, $i = 1, \ldots, M$, that can be described by the following equation

$$e_t^j = a^{i,j} \cdot t + b^{i,j}, \quad t \in I^i = \{s \in \mathbb{N}_0 \,|\, t_0 + 5(i-1) < s \leq t_0 + 5\,i\}, \tag{11}$$

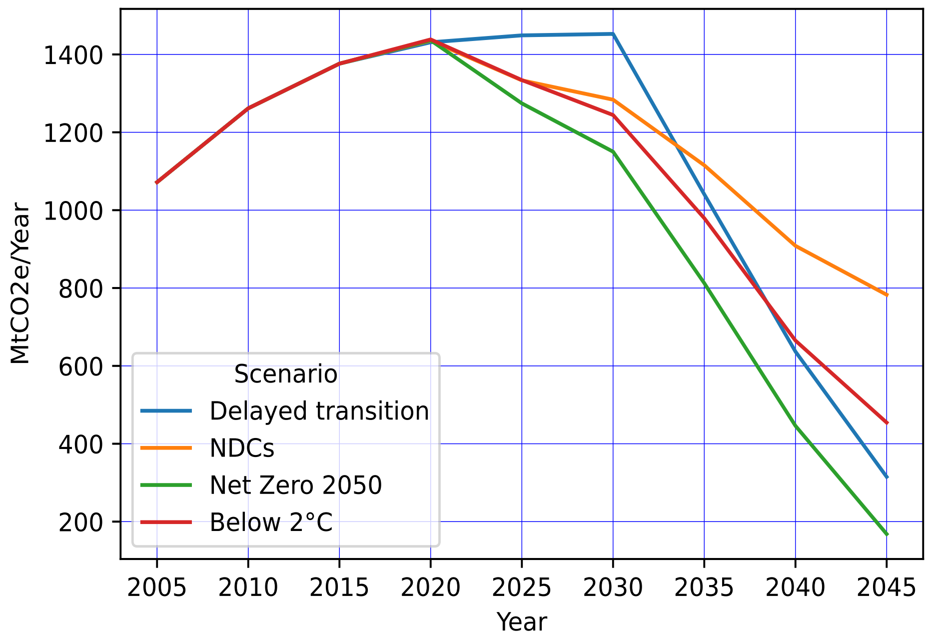


Figure 2: Selected Scenarios for Chemicals in GCAM 5.3+ Model provided by NGFS. Source: IIASA Scenario Explorer.

where t_0 denotes the base year, $a^{i,j}$ and $b^{i,j}$ are the slope and the intercept in the i-th linear period of the j-th scenario at time t. Moreover, we assume that $e_{t_0} > 0$.

Let us denote a vector of annual decarbonization rates of a scenario j at the beginning of each linear period by $(R^j_{5(i-1)+1})_{i=1,\ldots,M}$. Then, to determine the calibrated carbon emissions value at time $t = 1$ in a scenario j, we apply the first scenario decarbonization rate, R^j_1, to the base year's emissions e_{t_0} and obtain that

$$a^{1,j} = e_{t_0}(1 - R^j_1) - e_{t_0} \quad \text{and} \quad b^{1,j} = e_{t_0}(1 - R^j_1) - a^{1,j}\,(t_0 + 1).$$

As linear increments are constant within each linear period, the first linear period can be described by $e^j_t = a^{1,j}t + b^{1,j}$ for t with $t_0 < t \leq t_0 + 5$. We proceed similarly using the last emission value of a previous linear period and an appropriate scenario decarbonization rate to obtain slopes and intercepts for all linear periods in all scenarios. See Figure 4 for selected calibrated

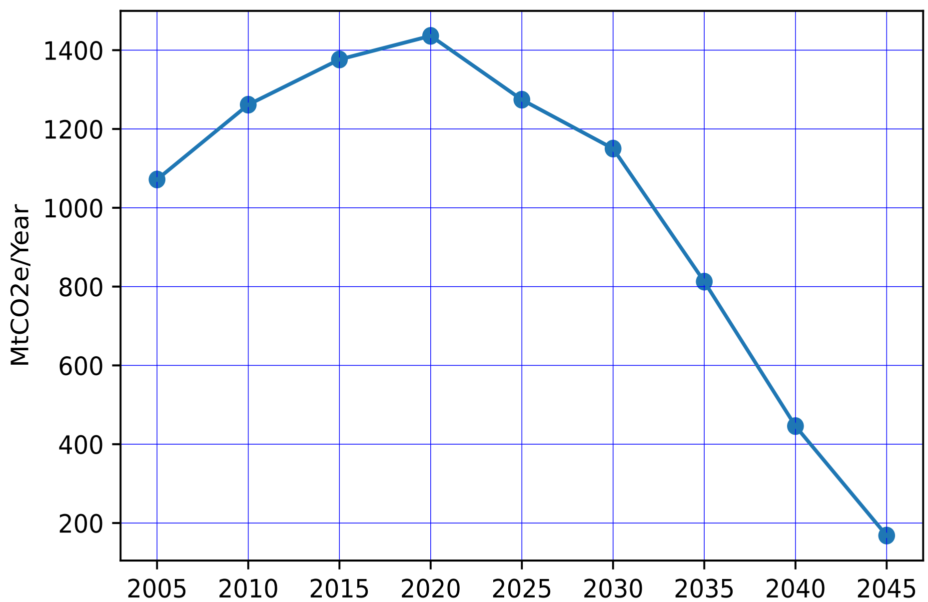


Figure 3: A piecewise linear structure of the net-zero scenario for Chemicals with an interval length of five years in GCAM 5.3+ Model. Source: IIASA Scenario Explorer.

scenarios for Solvay SA, a listed company located in Belgium and classified as a chemical company with the base year of 2018.
Note that we consider carbon emissions up to the time point of net-zero $t \in (t_0, T]$, where $T := T(j, s)$ is the time when net-zero emissions are reached, which depends on the scenario $j \in J$ and the sector $s \in S$ with S denoting the set of sectors under consideration. Consequently, for calibrated carbon emissions of a scenario j holds that $e_t^j > 0$ for $t \in (t_0, T]$.

3.2. Inclusion of Net-Zero Targets

Additionally to a general scenario calibration, we can adjust the calibrated net-zero emission pathway to include emission reduction targets set by a company. This would imply an emission pathway that not only includes sectoral decarbonization abilities but also incorporates individual forward-looking transition plans. For instance, a firm can expect to get a future transition-relevant innovation later or sooner than the sector average and,

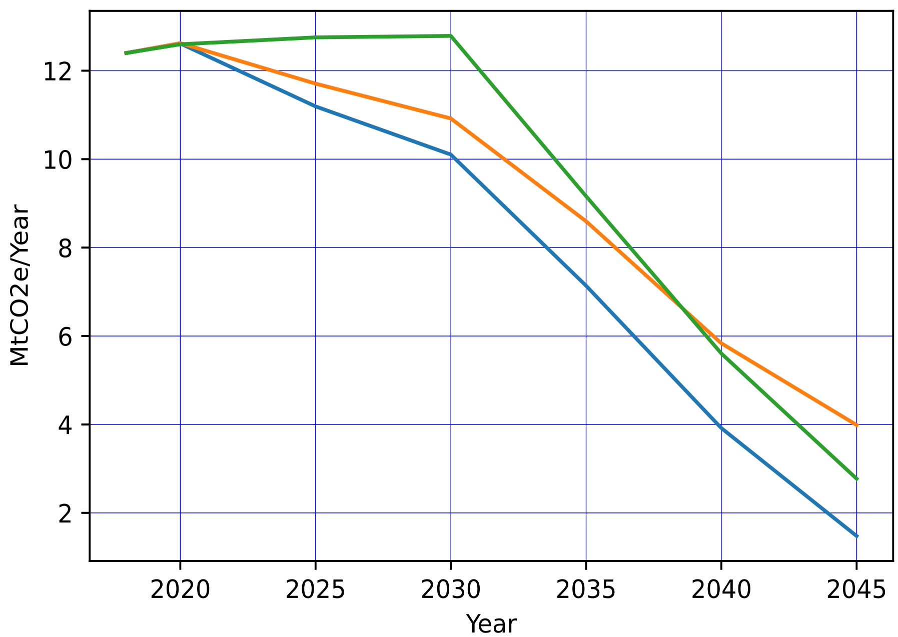


Figure 4: Calibrated net-zero, well-below 2°C and delayed transition scenarios for Solvay, a German chemicals company.

therefore, deviate from the scenario pathway.

For a net-zero scenario, we define an emission pathway with varying length of linear periods

$$\tilde{e}_t = a^i\, t + b^i, \quad t \in \tilde{I}^i = \{s \in \mathbb{N}_0 \,|\, t_0 + \sum_{r=1}^{i-1} \tau^r < s \leq t_0 + \sum_{r=1}^{i} \tau^r\},$$

where $\tau^i \geq 0$ denotes the length of the i-th linear period and it holds that $\sum_i \tau^i = T - t_0 + 1$. We determine the length of each linear period, so that to approximate emission targets at corresponding target times

$$\tilde{e}_t = g^{k(t)} > 0, \quad t \in G$$

where $G \subset [t_0, T]$ is a set of target times, $k(t)$ assigns each target time a target number and $g^{k(t)}$ is the target emissions amount at time t. In the case of Solvay, a possible target-inclusive net-zero scenario is illustrated in Figure

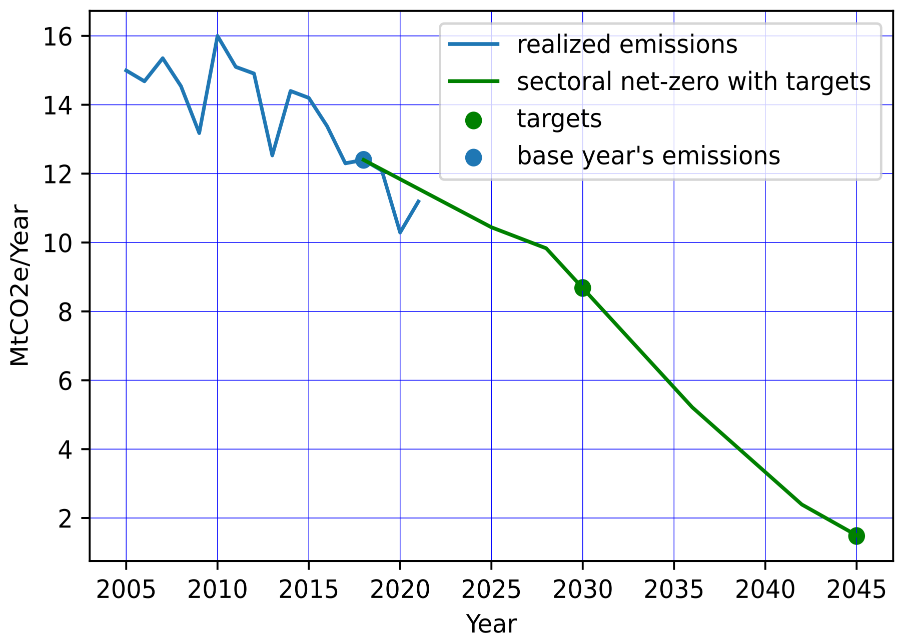


Figure 5: Calibrated net-zero scenario that includes a near-term in 2030 and a long-term target in 2045 of Solvay.

5.
Note that $\tilde{e}_t > 0$ for $t \in [t_0, T]$ as per construction the last target's time is at the terminal time T, where $g^{k(T)} > 0$, and calibrated net-zero emission pathway is decreasing.
Moreover, we introduce the carbon budget implied by the calibrated net-zero emissions pathway:

$$\tilde{b}\,(t_0, T) := \sum_i \int_{\tilde{I}^i} \tilde{e}_t \, dt > 0.$$

4. Empirical Analysis

In this Section, we present the range of estimated model parameters for companies which already started decarbonizing in 2016. Furthermore, we apply our model for two simulated emission paths and compute the probabilities of staying below the calibrated net-zero emission pathway and the corresponding carbon budget. We also analyze the performance of several companies from Chemicals, Elictricity and Real Estate sectors.

4.1. Parameter Estimation

We retrieve corporate emissions data from Refinitiv ESG Database and preprocess it to obtain Scope 1 and Scope 2 emissions data, which correspond to direct and indirect emissions, respectively. The latter are specifically indirect emissions coming from the generation of purchased energy. We do not consider Scope 3 emissions as they are not reflected in sectoral scenarios and too little emissions data is available on it.

Emissions data from 2016 to 2022 is considered as it is the greatest time range when the majority of companies has started to decarbonize and the data is already available. In total, we obtain 1408 companies that decarbonize in this time range. Using the observed emissions data, we estimate the parameters for carbon emissions, which are the mean emission reduction rate, R, and the standard deviation of emission reduction rates, σ, see Equation (2). To compute the standard deviation, we first detrend emission reduction rates and then compute the standard deviation with the normalization factor of $N - 1$, where N denotes the number of observations. Figures C.8 and C.9 show 5-95%-percentile parameter ranges for R and σ. The values of R range from around 0.01 to 0.3, while for σ the corresponding range is 0.02 to 0.3, both considering values within 5-th to 95-th percentile.

4.2. Example of Model Application

We next provide an example of computing the probability of staying below or at the calibarted carbon emission pathway as well as the probability of respecting the corresponding carbon budget. For this, we use the calibrated net-zero emission pathway of a chemical company Solvay SA, see also Figure 5.

The probability of staying below the calibrated net-zero emission pathway over the net-zero transition period $[t_0, T]$ reads

$$\mathbb{P}\left(\mathcal{CE}(t) \leq \tilde{e}(t)\right) = \Phi\left(\frac{\log(\tilde{e}(t)) - m(t)}{v(t)}\right), \tag{12}$$

where we recall that $\tilde{e}(t)$ is the calibrated net-zero emission pathway including targets and $m(t)$, $v(t)$ are the log-normal parameters of carbon emissions.

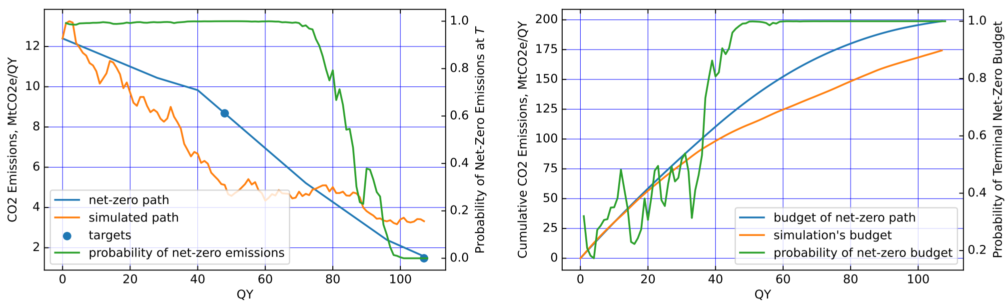
To determine the probability of respecting the carbon budget implied by the calibrated net-zero emissions pathway at time $s \in [t_0, T]$, we compute

$$\mathbb{P}\left(\mathcal{CB}(t_0,T) \leq \tilde{b}(t_0,T)\right) = \Phi\left(\frac{\log(\tilde{b}(t_0,T) - \mathcal{CB}(t_0,s)) - \bar{\mu}(s,T)}{\bar{v}(s,T)}\right), \qquad (13)$$

where $\tilde{b}(t_0,T)$ is the carbon budget implied by the calibrated net-zero emission pathway over the whole net-zero transition period $[t_0,T]$ and $\bar{\mu}(s,T)$, $\bar{v}(s,T)$ are log-normal parameters of the carbon budget's approximation over the time period $[s,T]$.

We next simulate an emissions path using estimated parameters, the base year and the base year's emissions of Solvay SA. We also use the calibrated net-zero emission pathway of Solvay SA, see Figure 5. In Subfigures 6(a) and 7(a), we compute the probability of staying below the calibrated net-zero emission pathway at each time given by Equation (12). The probabilities of achieving the targets can be identified at the target times $t \in G$, as the targets are included in the calibrated emission pathway. Moreover, in Subfigures 6(b) and 7(b), we compute the probability of respecting the carbon budget implied by the calibrated net-zero emissions pathway, given in Equation (13).

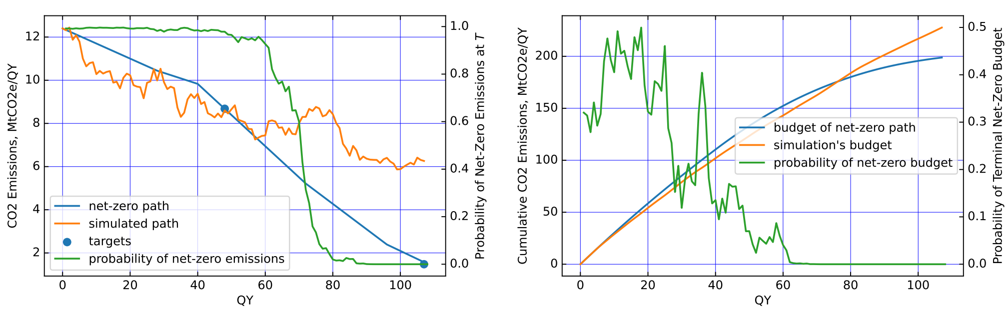
Probabilities of staying below or at the calibrated net-zero emission pathway behave in a similar way for both simulated emission paths with staying in high probability ranges as the simulated path is below the calibrated pathway and decreasing when it starts to deviate upwards. Note that even though the simulated path in Subfigure 6(a) produces less excess emissions than the path in Subfigure 6(b), the probabilities of reaching net-zero at the end of the transition period are both approximately zero. While, considering the probabilities of respecting the corresponding carbon budget, we have a slightly different situation, where the overall relation of accumulated emissions of the simulated pathway to these of the calibrated scenario pathway impacts the probabilities. Thus, in Subfigure 6(a), the probability of respecting the implied calibrated net-zero emissions pathway approximately reaches the value of one after 40 observations, whereas in Subfigure 6(b) it is zero after approximately 60 observations. Here, we can again observe the advantage of considering the carbon budget instead of carbon emissions as it enables an analysis of accumulated emissions relative to a benchmark and, thus, provides an overall analysis of the net-zero transition.



(a) Emissions

(b) Budget

Figure 6: (a): Simulated Emission Path with $R \approx 0.3$ and $\sigma \approx 0.08$ and the Probability of Staying Below Calibrated Net-Zero Emissions; (b): Carbon Budget of the Simulated Emissions Path, the Probability of Respecting the Remaining Carbon Budget Implied by Calibrated Net-Zero Emission Pathway.



(a) Emissions

(b) Budget

Figure 7: (a): Simulated Emission Path with $R \approx 0.3$ and $\sigma \approx 0.08$ and the Probability of Staying Below Calibrated Net-Zero Emissions; (b): Carbon Budget of the Simulated Emissions Path, the Probability of Respecting the Remaining Carbon Budget Implied by Calibrated Net-Zero Emission Pathway.

4.3. Sectoral Analysis

In addition to the example of model application given above, we analyze several firms of different sectors to understand the scale of firm's targets and ambitions as well as the decarbonization performance towards these. Specifically, for a couple of firms operating in the sector of Chemicals, Real Estate and Electricity, we provide the time series of probabilities of reaching their targets as well as respecting the implied carbon budget at the corresponding target times.

The results are provided in Tables B.4, B.5, B.6, B.7, B.8 and B.9 in Appendix. Moreover, the estimated parameters used in the analysis of firms are given in Table B.10. Table B.11 presents emission targets and targets years retrieved from the Target Dashboard by the SBTi, see SBTi (2023). While in Table B.12 the carbon budget up to a corresponding target is given.

In Real Estate sector we considered Cushman & Wakefield PLC, Tokyu Fudosan Holdings Corp and Shui On Land Limited, which all have a near-term target assessed as 1,5°C-aligned by the SBTi. Cushman & Wakefield PLC has in addition a long-term net-zero target in 2050 also assessed as 1,5°C-aligned by the SBTi, whereas Tokyu Fudosan Holdings Corp has committed a net-zero target to the SBTi and Shui On Land Limited solely states its willingness to reach net-zero in its sustainability commitment. Cushman & Wakefield PLC, showing a historical mean emission reduction rate of around 10% and the volatility of 5%, will very likely reach its emission targets and the corresponding carbon budget considering the assessed probabilities. Tokyu Fudosan Holdings Corp has a slightly higher emission reduction rate and a lower volatility than Shui On Land Limited. Here, it is interesting that the probabilities are quite similar either considering carbon emissions or carbon budget in 2019 and 2020. While in 2021, after a major emission reduction of over 20% by Shui On Land Limited, the probabilities for reaching emission targets and respecting the corresponding carbon budget turn out to be higher than these of Tokyu Fudosan Holdings Corp.

The firms from Chemicals sector that we analyzed are Solvay SA, FMC Corp and Johnson Matthey PLC. All firms have a near-term target approved by the SBTi. The near-term target of FMC Corp is assessed to be 1,5°C-aligned and the targets of Solvay SA and Johnson Matthey PLC are assessed to be well below 2°C-aligned. Considering the long-term net-zero targets, FMC Corp has a net-zero target in 2035, which is approved by the SBTi to be 1,5°C-aligned, Johnson Matthey PLC has commited a net-zero target to the SBTi and Solvay SA states its willingness to reach net-zero. FMC Corp shows the greatest historical mean emission reduction rate among the considered companies in the sector, which is of about 11% . Nevertheless, it has a high volatility of almost 15%. Solvay SA and Johnson Matthey PLC have similar parameters with Solvay SA having a slightly greater mean emission reduction rate and volatility. From the probabilities, we can clearly see that FMC Corp, having the probability of about 90% of respecting the carbon budget up to both target times, will very likely succeed to deliver the pledged net-zero transition, even though the probability of reaching net-zero emissions is about 5%. For Solvay SA the probabilities of reaching near-term target's emissions and of respecting the carbon budget up to the near-term target year are of about 60% and 75% respectively in 2021. Despite this, Solvay SA is highly unlikely to reach the net-zero target in terms of emissions, but it still has a probability of about 30% in 2021 to respect the carbon budget up to the net-zero target year. Johnson Matthey PLC, however, still continues to increase its emissions and, thus, has very low probabilities both in terms of carbon emissions and the carbon budget.

We also provide the assessment of two companies from Electricity sector: Contact Energy Limited and Iberdrola SA. Both have a near-term target assessed by the SBTi to be 1,5°C-aligned. Iberdrola SA has in addition a net-zero target in 2039 assessed by the SBTi as 1,5°C-aligned, while Contact Energy Limited solely states its willingness to reach net-zero. Concerning the used parameters, Iberdrola SA has a mean historical emission reduction rate of around 10% and a rather small volatility of 5%, whereas for Contact Energy Limited these are about 5% and 10%, respectively. Even though Iberdrola SA has slightly increased its emissions from 2020 to 2021, it still has a high probability of respecting the pledged carbon budget up to the first and the second target. The probability of reaching the near-term target in terms of emissions is about 87%, while the probability of reaching net-zero emissions is about 25%. Contact Energy Limited constantly reduces its carbon emissions with slight fluctuations following its base year of 2018. The probability of reaching the near-term target and respecting the corresponding carbon budget up to this target are 71% and 97% as of 2022, respectively. However, the corresponding probabilities for the net-zero target are noticeably lower.

All in all, we can observe that among the selected firms, the firms with approved near- and long-term targets by the SBTi have performed better in the past 5 to 7 years and show higher historical mean emission reduction rates. All of them are very likely to respect the carbon budget implied by the calibrated net-zero emission pathway at target times and, thus, to preserve the pledged total amount of carbon emissions. Most of the companies with only a clearly stated near-term target show high probabilities of respecting the carbon budget implied by the calibrated net-zero scenario up to the target time, whereas such probabilities for a net-zero target turned out to be rather small to moderate. Furthermore, as we have seen, better model parameters do not necessarily imply higher probabilities, as the probabilities are adjusted with each new observation after the base year. It means that companies can still increase their probabilities, provided the overall carbon budget implied by the calibrated net-zero emission pathway has not been exceeded. It highlights the importance of timely regulation that would oversee the time series of probabilities of respecting the carbon budget implied by the calibrated net-zero emission pathway and adjust future emission reduction rates to obtain a desired probability value.

5. Conclusion

In order to limit the global average temperature rise as much as possible, we need to stick to an emission scenario ensuring driving carbon emissions to the level where they can be completely compensated by carbon removals, so that we arrive at the net carbon emissions amount of zero. IPCC's SSP1-1.9 scenario, for example, reaches net-zero carbon emissions by 2050 and limits the long-term average temperature rise to the range of 1°C to 1.8°C, see IPCC (2021). Moreover, to ensure a high likelihood of achieving the temperature limit goal of 1.5°C, it is crucial to preserve a corresponding carbon budget, which is the cumulative amount of carbon emissions over the transition period. For instance, the global carbon budget of 300 GtCO2e counting from the beginning of 2020 provides the likelihood of 83% of limiting global warming to 1.5 °C, see IPCC (2021). In our approach, we assess the probability of reaching emission targets and following the calibrated net-zero emission pathway, but a great emphasis is put on probabilities of respecting the corresponding carbon budget. Not only is the cumulative amount of emissions during the transition necessary to ensure a specific certainty of future implied global average temperature rise, but it also provides a method of assessment of the overall transition.

In our work, we specifically focus on firms that stated their willingness to reach net-zero emissions. We are convinced that an individual analysis is required for a specific and more effective regulation. Moreover, we provide a probabilistic modelling approach for firms' projected carbon emissions and carbon budget that incorporates the inherent variability of emission reductions.

As we can see there is an increasing tendency of setting emissions targets, including net-zero emission targets, see for instance an evolution of the SBTi or other similar initiatives. In order to approximate a pledged carbon emission pathway, we incorporate pledged emission targets into a calibrated net-zero scenario pathway that also considers sectoral abilities to decarbonize. We next assess the performance of firms with respect to the calibrated net-zero emission pathway. Moreover, we compute the carbon budget implied by the calibrated emission scenario pathway.

We model the projected carbon emissions of the firm as a gBM with the drift and the volatility based on historical emission reduction rates. The projected carbon budget is defined as an integral over the projected carbon emissions and is approximated with a log-normal distribution by moment matching.

When applying our model we can observe how different the probability of reaching net-zero emissions and the probability of respecting the corresponding carbon budget behave. While the former can be almost zero, the latter one can be quite high. Which means that failing targets does not necessarily mean failing the transition as it can still be possible to preserve a pledged cumulative emissions amount. It emphasizes the importance of considering carbon budget that allows us to make statements on the overall net-zero transition, not just on carbon emissions amount at several time points.

Even though firms under consideration with both near- and long-term targets performed excellent in terms of all probabilities. We still observed firms that have a near-term target, but that have a pretty low probability of achieving it or respecting the pledged carbon budget up to the target time. Which means that setting an emission reduction target does not imply a successful transition to it. Consequently, more attention should be drawn to the transition plan that is might be reflected in company's annual reports, capital allocation, investments and transition management to ensure that the firm will indeed implement the pledge.

Moreover, during the assessment of companies from different sectors, we also noticed that firms with better historical performance can still be outperformed by firms which perform better following the base year. As we dynamically adjust the probabilities with each new observation, for instance, the probability of respecting the pledged carbon budget can improve. Therefore, overseeing and regulating emission reduction rates in order to improve the probability of respecting the pledged carbon budget is crucial to preserve resilience towards climate risks.

Even though our modelling approach relies on many assumptions and approximations, it is flexible and can be adjusted to meet the needs of the user. For instance, it is possible to use different scenarios and update emission reduction targets if necessary. The assumption of a constant negative drift of carbon emission process can be replaced by a non-constant one allowing more flexibility for making a more tailored emission projections. Lastly, expert users could adjust the resulting probabilities to incorporate their intuition on the scale of the implementation of the pledge.

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Appendix A. Mathematical Results

1. We first show that $\mathbb{E}[CB(t_0,T)] > 0$. We recall that for $t_0 < T$, the first moment of the carbon budget reads

$$\mathbb{E}[\mathcal{CB}(t_0,T)] = \frac{\mathcal{CE}(t_0)}{\log(1-R)} \left(\exp(\log(1-R)(T-t_0)) - 1\right),$$

where $\mathcal{CE}(t_0) > 0$ and $0 < R < 1$. The latter implies that $(1-R) \in (0,1)$ and, thus, that $\log(1-R) < 0$ as well as $\exp(\log(1-R)(T-t_0)) \in (0,1)$, which ends the proof.

2. Second, we show that $\mathbb{E}[\mathcal{CB}(t_0,T)^2] > 0$. Again, recall that the second moment is given by

$$\mathbb{E}[\mathcal{CB}^2(t_0,T)] = \mathcal{CE}^2(t_0)\Bigg(\frac{\exp\left((2\,m+\sigma^2)(T-t_0)\right)}{(m+\sigma^2/2)(m+\sigma^2)} - \frac{2\exp\left(m(T-t_0)\right)}{m(m+\sigma^2)} + \frac{1}{m(m+\sigma^2/2)}\Bigg),$$

where we use the following shorthand notation $m := \log(1-R)$.
We consider the following three cases:
   a) $\log(1-R) + \sigma^2/2 > 0 \implies \log(1-R) + \sigma^2 > 0$,
   b) $\log(1-R) + \sigma^2 < 0 \implies \log(1-R) + \sigma^2/2 < 0$,
   c) $\log(1-R) + \sigma^2 > 0$ and $\log(1-R) + \sigma^2/2 < 0$.

In each case, we show that the second moment is strictly positive:
   a) : for the first two terms, considering our assumption in a), we have that

$$\frac{\exp\left((2\,m+\sigma^2)(T-t_0)\right)}{(m+\sigma^2/2)(m+\sigma^2)} > 0 \quad \text{and} \quad -\frac{2\exp\left(m(T-t_0)\right)}{m(m+\sigma^2)} > 0,$$

while, for the remaining term, it holds that

$$\frac{1}{m(m+\sigma^2/2)} < 0.$$

We, thus, show that

$$\frac{\exp\left((2\,m+\sigma^2)(T-t_0)\right)}{(m+\sigma^2/2)(m+\sigma^2)} - \frac{2\exp\left(m(T-t_0)\right)}{m(m+\sigma^2)} > \frac{-1}{m(m+\sigma^2/2)}.$$

We rewrite the left-hand side as follows

$$\begin{aligned}
&\frac{\exp((2\,m+\sigma^2)(T-t_0))m - 2\exp(m(T-t_0))(m+\sigma^2/2)}{m(m+\sigma^2/2)(m+\sigma^2)} \\
\geq\ &\frac{\exp(m(T-t_0))(m-2m-\sigma^2)}{m(m+\sigma^2/2)(m+\sigma^2)} \Leftrightarrow \frac{-\exp(m(T-t_0))}{m(m+\sigma^2/2)}.
\end{aligned}$$

What remains to show is that

$$-\exp(m(T-t_0)) > -1 \Leftrightarrow \exp(m(T-t_0)) < 1,$$

which holds as $m < 0$.

b) : due to the assumption in b), we obtain that

$$\frac{\exp\left((2\,m+\sigma^2)(T-t_0)\right)}{(m+\sigma^2/2)(m+\sigma^2)} > 0 \quad \text{and} \quad \frac{1}{m(m+\sigma^2/2)} > 0.$$

as well as

$$-\frac{2\exp\left(m(T-t_0)\right)}{m(m+\sigma^2)} < 0.$$

In the same vein, we show that

$$\frac{\exp\left((2\,m+\sigma^2)(T-t_0)\right)}{(m+\sigma^2/2)(m+\sigma^2)} + \frac{1}{m(m+\sigma^2/2)} > \frac{2\exp\left(m(T-t_0)\right)}{m(m+\sigma^2)}.$$

For the left-hand side, we obtain

$$\frac{\exp\left((2\,m+\sigma^2)(T-t_0)\right)}{(m+\sigma^2/2)(m+\sigma^2)} + \frac{1}{m(m+\sigma^2/2)} > \frac{\exp\left(m(T-t_0)\right)}{(m+\sigma^2/2)(m+\sigma^2)},$$

as the second term on the left hand side is strictly positive. Furthermore, we have

$$\begin{aligned}
&\exp\left(m(T-t_0)\right)m - 2\exp\left(m(T-t_0)\right)(m+\sigma^2) \\
\Leftrightarrow\ &-\exp\left(m(T-t_0)\right)(m+\sigma^2) > 0,
\end{aligned}$$

where the last inequality holds as we assumed $m+\sigma^2 < 0$ in b). It finishes the proof for the b).

c) : the assumptions of the c) imply that

$$-\frac{2\exp\left(m(T-t_0)\right)}{m(m+\sigma^2)} > 0 \quad \text{and} \quad \frac{1}{m(m+\sigma^2/2)} > 0.$$

as well as

$$\frac{\exp\left((2\,m+\sigma^2)(T-t_0)\right)}{(m+\sigma^2/2)(m+\sigma^2)} < 0.$$

We, therefore, show that

$$-\frac{2\exp\left(m(T-t_0)\right)}{m(m+\sigma^2)} + \frac{1}{m(m+\sigma^2/2)} > \frac{-\exp\left((2\,m+\sigma^2)(T-t_0)\right)}{(m+\sigma^2/2)(m+\sigma^2)},$$

where the left-hand side can be rewritten as

$$\begin{aligned}
& -\frac{2\exp\left(m(T-t_0)\right)}{m(m+\sigma^2)} + \frac{1}{m(m+\sigma^2/2)} \\
\geq\ & \frac{-2\exp\left((2\,m+\sigma^2)(T-t_0)\right)(m+\sigma^2/2)+(m+\sigma^2)}{m(m+\sigma^2/2)(m+\sigma^2)} \\
\geq\ & \frac{(m+\sigma^2/2)(1-2\exp\left((2\,m+\sigma^2)(T-t_0)\right))}{m(m+\sigma^2/2)(m+\sigma^2)} \\
\geq\ & \frac{(1-2\exp\left((2\,m+\sigma^2)(T-t_0)\right))}{(m+\sigma^2/2)(m+\sigma^2)}.
\end{aligned}$$

It remains to show that

$$\begin{aligned}
& 1-2\exp\Big((2\,m+\sigma^2)(T-t_0)\Big) > -\exp\Big((2\,m+\sigma^2)(T-t_0)\Big) \\
\Leftrightarrow\ & \exp\Big((m+\sigma^2/2)2(T-t_0)\Big) < 1,
\end{aligned}$$

which holds as in c) we assumed that $m+\sigma^2/2<0$.

Appendix B. Tables

σ	R	Approx.	MC	St. Error	Difference
0.05	0.01	0.314756	0.315795	$6 \cdot 10^{-6}$	-0.001
	0.03	0.618765	0.621275	$7 \cdot 10^{-6}$	-0.0025
	0.07	0.973984	0.972625	$3 \cdot 10^{-6}$	0.0014
0.1	0.01	0.423648	0.427765	$5 \cdot 10^{-6}$	-0.0041
	0.03	0.578682	0.584195	$7 \cdot 10^{-6}$	-0.0055
	0.07	0.845249	0.845605	$4 \cdot 10^{-6}$	-0.0004
0.2	0.01	0.500696	0.510020	$5 \cdot 10^{-6}$	-0.0093
	0.03	0.577234	0.587445	$7 \cdot 10^{-6}$	-0.0102
	0.07	0.725263	0.733745	$5 \cdot 10^{-6}$	-0.0085
	0.15	0.931442	0.929565	$3 \cdot 10^{-6}$	0.0019
0.3	0.01	0.544545	0.557840	$5 \cdot 10^{-6}$	-0.0133
	0.03	0.594007	0.608290	$7 \cdot 10^{-6}$	-0.0143
	0.07	0.691912	0.706120	$4 \cdot 10^{-6}$	-0.0142
	0.15	0.860134	0.866280	$3 \cdot 10^{-6}$	-0.0061
	0.20	0.931304	0.931340	$3 \cdot 10^{-6}$	0

Table B.1: Absolute error comparison with Monte Carlo, MC, simulation with 200000 iterations and $N = 20$ observations.

σ	R	Approx.	MC	St. Error	Difference
0.05	0.01	0.199505	0.198970	$6 \cdot 10^{-6}$	0.0005
	0.03	0.648130	0.652505	$6 \cdot 10^{-6}$	-0.0044
	0.07	0.998608	0.998335	$3 \cdot 10^{-7}$	0.0003
0.1	0.01	0.365385	0.369915	$8 \cdot 10^{-6}$	-0.0045
	0.03	0.603680	0.610955	$7 \cdot 10^{-6}$	-0.0073
	0.07	0.940381	0.938055	$3 \cdot 10^{-6}$	0.0023
0.2	0.01	0.494461	0.503600	$8 \cdot 10^{-6}$	-0.0091
	0.03	0.610216	0.625190	$6 \cdot 10^{-6}$	-0.0150
	0.07	0.817817	0.827585	$5 \cdot 10^{-6}$	-0.0098
	0.15	0.990638	0.988140	$1 \cdot 10^{-6}$	0.0025
0.3	0.01	0.568939	0.580855	$6 \cdot 10^{-6}$	-0.0119
	0.03	0.640097	0.658780	$6 \cdot 10^{-6}$	-0.0187
	0.07	0.774544	0.795005	$5 \cdot 10^{-6}$	-0.0205
	0.15	0.952467	0.953720	$2 \cdot 10^{-6}$	-0.0013
	0.20	0.990378	0.988050	$1 \cdot 10^{-6}$	0.0023

Table B.2: Absolute error comparison with Monte Carlo, MC, simulation with 200000 iterations and $N = 50$ observations.

σ	R	Approx.	MC	St. Error	Difference
0.05	0.01	0.004832	0.003495	10^{-6}	0.0013
	0.03	0.173878	0.172405	$5 \cdot 10^{-6}$	0.0015
	0.07	0.996423	0.995350	$5 \cdot 10^{-6}$	0.0011
0.1	0.01	0.120550	0.109425	$5 \cdot 10^{-6}$	0.0111
	0.03	0.358445	0.364715	$9 \cdot 10^{-6}$	-0.0063
	0.07	0.922531	0.923130	$5 \cdot 10^{-6}$	-0.0006
0.2	0.01	0.369271	0.359050	$9 \cdot 10^{-6}$	0.0102
	0.03	0.516527	0.526170	$9 \cdot 10^{-6}$	-0.0096
	0.07	0.808735	0.826265	$6 \cdot 10^{-6}$	-0.0175
	0.15	0.998134	0.996375	$6 \cdot 10^{-7}$	0.0018
0.3	0.01	0.531832	0.510580	10^{-5}	0.0213
	0.03	0.617085	0.620060	$8 \cdot 10^{-6}$	-0.0030
	0.07	0.782753	0.809540	$6 \cdot 10^{-6}$	-0.0268
	0.15	0.978269	0.978715	$2 \cdot 10^{-6}$	-0.0004
	0.20	0.998364	0.996745	$5 \cdot 10^{-7}$	0.0016

Table B.3: Absolute error comparison with Monte Carlo, MC, simulation with 200000 iterations and $N = 108$ observations.

Name	Year	Realized Emissions	$p\left(\mathcal{CE}; g^1\right)$	$p\left(\mathcal{CE}; g^2\right)$
Cushman & Wakefield PLC	2019	0.0369	0.9999	1
	2020	0.0364	0.9996	1
	2021	0.0331	0.9997	1
Tokyu Fudosan Holdings Corp	2019	0.2305	0	0.0060
	2020	0.2195	0	0.0085
	2021	0.2064	0	0.0160
Shui On Land Limited	2019	0.102	0.0394	0.2295
	2020	0.1063	0.0152	0.1740
	2021	0.083	0.1045	0.3508

Table B.4: Time series of probabilities of staying below or at target emission level, $g^1 > 0$ and $g^2 > 0$, at corresponding target times, which here are denoted by $p\left(\mathcal{CE}; g^1\right)$ and $p\left(\mathcal{CE}; g^2\right)$. The probabilities are given for all available emissions observations following the base year for three companies of Real Estate sector.

Name	Year	Realized Budget	$p\left(\mathcal{CB}; g^1\right)$	$p\left(\mathcal{CB}; g^2\right)$
Cushman & Wakefield PLC	2019	0	0.9997	1
	2020	0.0366	0.9958	1
	2021	0.0714	0.9980	1
Tokyu Fudosan Holdings Corp	2019	0	0.0210	0.0091
	2020	0.225	0.0333	0.0143
	2021	0.438	0.1133	0.0379
Shui On Land Limited	2019	0	0.1288	0.0683
	2020	0.1041	0.0261	0.0249
	2021	0.1988	0.3560	0.1629

Table B.5: Time series of probabilities of respecting the carbon budget implied by the net-zero emission pathway up to the first and the second target times, which here are denoted by $p\left(\mathcal{CB}; g^1\right)$ and $p\left(\mathcal{CB}; g^2\right)$. The probabilities are given for all available carbon budget observations following the base year for three companies of Real Estate sector.

Name	Year	Realized Emissions	$p\,(\mathcal{CE};T^1)$	$p\,(\mathcal{CE};T^2)$
Solvay SA	2018	12.40	0.5983	0.0016
	2019	12.09	0.5873	0.0012
	2020	10.29	0.7755	0.0029
	2021	11.19	0.6082	0.0008
FMC Corp	2021	0.1659	0.9168	0.0456
Johnson Matthey PLC	2020	0.451882	0.0006	0
	2021	0.4313	0.0017	0
	2022	0.4607	0.0000	0

Table B.6: Time series of probabilities of staying below or at target emission level, g^1 and g^2, at corresponding target times, which here are denoted by $p\,(\mathcal{CE};g^1)$ and $p\,(\mathcal{CE};g^2)$. The probabilities are given for all available emissions observations following the base year for three companies of Chemicals sector.

Name	Year	Realized Budget	$p\,(\mathcal{CB};g^1)$	$p\,(\mathcal{CB};g^2)$
Solvay SA	2018	0	0.6463	0.3235
	2019	12.245	0.6426	0.2951
	2020	23.435	0.9236	0.5315
	2021	34.175	0.7570	0.2950
FMC Corp	2021	0	0.9237	0.8979
Johnson Matthey PLC	2020	0	0.2210	0.0000
	2021	0.4416	0.5037	0.0000
	2022	0.8876	0.0001	0.0000

Table B.7: Time series of probabilities of respecting the carbon budget implied by the net-zero emission pathway up to the first and the second target times, which here are denoted by $p\,(\mathcal{CB};g^1)$ and $p\,(\mathcal{CB};g^2)$. Th probabilities are given for all available carbon budget observations following the base year for three companies of Chemicals sector.

Name	Year	Realized Emissions	$p\left(\mathcal{CE}; g^1\right)$	$p\left(\mathcal{CE}; g^2\right)$
Contact Energy Limited	2018	1.1772	0.4404	0.0082
	2019	0.9873	0.5900	0.0127
	2020	0.9243	0.6114	0.0116
	2021	1.0460	0.3560	0.0037
	2022	0.7882	0.7105	0.0109
Iberdrola SA	2020	15.1380	0.9815	0.5397
	2021	15.3690	0.8656	0.2514

Table B.8: Time series of probabilities of staying below or at target emission level, g^1 and g^2, at corresponding target times, which here are denoted by $p\left(\mathcal{CE}; g^1\right)$ and $p\left(\mathcal{CE}; g^2\right)$. The probabilities are given for all available emissions observations following the base year for two companies of Electricity sector.

Name	Year	Realized Budget	$p\left(\mathcal{CB}; g^1\right)$	$p\left(\mathcal{CB}; g^2\right)$
Contact Energy Limited	2018	0	0.4737	0.2819
	2019	1.0822	0.7540	0.4313
	2020	2.0380	0.8325	0.4443
	2021	3.0232	0.4825	0.1978
	2022	3.9404	0.9700	0.4894
Iberdrola SA	2020	0	0.9979	0.9948
	2021	15.2535	0.8702	0.8473

Table B.9: Time series of probabilities of respecting the carbon budget implied by the net-zero emission pathway up to the first and the second target times, which here are denoted by $p\left(\mathcal{CB}; g^1\right)$ and $p\left(\mathcal{CB}; g^2\right)$. The probabilities are given for all available carbon budget observations following the base year for two companies of Electricity sector.

Name	Timeperiod	R	σ
Cushman & Wakefield PLC	2015-2022	0.1087	0.0484
Tokyu Fudosan Holdings Corp	2017-2021	0.0296	0.0205
Shui On Land Limited	2016-2021	0.0271	0.0706
Solvay SA	2016-2021	0.0318	0.0758
FMC Corp	2016-2021	0.1103	0.1477
Johnson Matthey PLC	2019-2022	0.0234	0.0156
Contact Energy Limited	2016-2022	0.047	0.1178
Iberdrola SA	2016-2021	0.1194	0.0437

Table B.10: The historical mean emission reduction rate R and the volatility of historical emission reductions σ assessed for selected firms based on the observations during the specified time range.

Name	Target Year	Target
Cushman & Wakefield PLC	2030	0.0184
	2050	0.0037
Tokyu Fudosan Holdings Corp	2030	0.1244
	2050	0.0674
Shui On Land Limited	2030	0.0383
	2050	0.0298
Solvay SA	2030	8.556
	2045	1.4771
FMC Corp	2030	0.0962
	2035	0.0166
Johnson Matthey PLC	2030	0.3028
	2045	0.0529
Contact Energy Limited	2026	0.6474
	2040	0.1071
Iberdrola SA	2030	5.2983
	2039	1.513805

Table B.11: Emission reduction targets and the corresponding target times for selected firms.

Name	Target Year	Target Budget
Cushman & Wakefield PLC	2030	0.3001
	2050	0.5739
Tokyu Fudosan Holdings Corp	2030	1.9987
	2050	4.0444
Shui On Land Limited	2030	0.8281
	2050	1.537
Solvay SA	2030	128.8692
	2045	199.1945
FMC Corp	2030	1.2421
	2035	1.5375
Johnson Matthey PLC	2030	3.9372
	2045	6.2392
Contact Energy Limited	2026	7.5907
	2040	13.0473
Iberdrola SA	2030	103.7428
	2039	132.7036

Table B.12: The carbon budget implied by the calibrated net-zero emission pathway at target times.

Appendix C. Figures

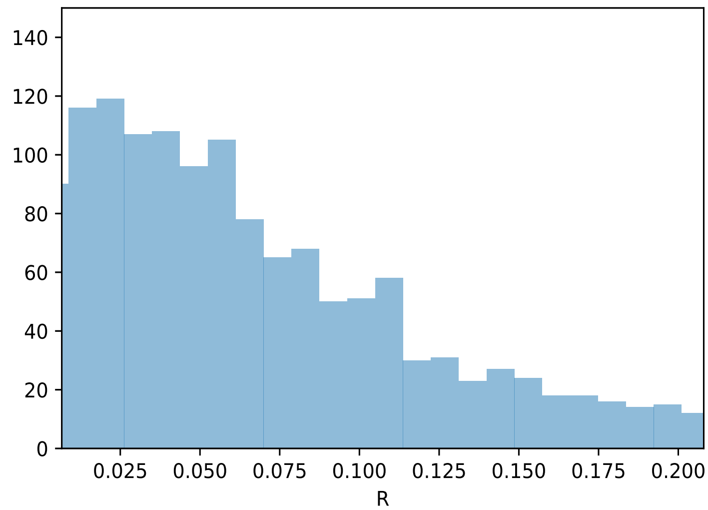

Figure C.8: 5-95% percentile value range for estimated R.

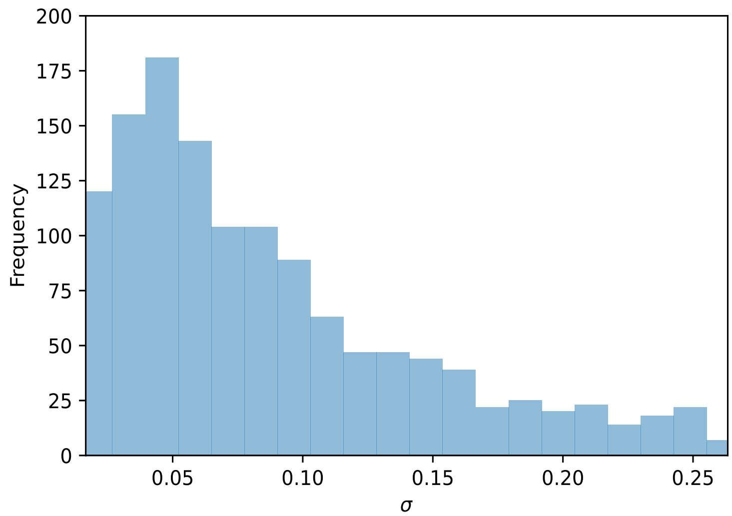

Figure C.9: 5-95% percentile value range for estimated σ.